PERFORMANCE ANALYSIS
OF A
HARTMANN WAVEFRONT SENSOR
USED FOR SENSING
ATMOSPHERIC TURBULENCE STATISTICS

Acknowledgements

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D_s $\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|u|$ L_0/D α . θ_0

F_{sm} $\vec{\rho}, \sigma_n, \vec{v}\tau i, \vec{p}_m^{(j)}, \vec{q}_m^{(j)}, \vec{p}_\mu^{(j)}, \vec{q}_\mu^{(j)}$

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. $|\vec{v}|$. D L_0/D ∞

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. $|\vec{v}|$. D L_0/D ∞

N $\vec{\rho}$. D, . $^\circ$ $\vec{v}$ α
. $|\vec{v}|$. D L_0/D ∞

N $\vec{\rho}$. D, . $^\circ$ $\vec{v}$ α
. $|\vec{v}|$. D L_0/D ∞

N $\vec{\rho}$. D, . $^\circ$ $\vec{v}$ α
. $|\vec{v}|$. D L_0/D ∞

N $\vec{\rho}$. D, . $^\circ$ $\vec{v}$ α
. $|\vec{v}|$. D L_0/D ∞

List of Tables

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \qquad \infty$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad \infty$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad .$$

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \qquad \infty$$

Abstract

PERFORMANCE ANALYSIS

OF A

HARTMANN WAVEFRONT SENSOR

USED FOR SENSING

ATMOSPHERIC TURBULENCE STATISTICS

I. Introduction

1.1 Background

seeing

r_0

seeing monitor

r_0

1.2 Atmospheric Turbulence

/

$\phi \; k$

$$\phi \; k \quad \pi^{\;(3-\alpha)} \quad \frac{}{\rho_0^{\alpha-2}} \quad \frac{c_1 \quad \frac{\alpha}{2}}{- \quad {}^{(4-\alpha)}\pi^2 \quad \frac{2-\alpha}{2}} \quad k^2 \quad {k_0}^2 \;\; {}^{-\frac{\alpha}{2}},$$

$$c_1 \quad \frac{}{\alpha -} \quad \frac{}{\alpha -} \quad {}^{\frac{\alpha-2}{2}},$$

$$\rho_0 \quad \frac{c_1 \quad \frac{\alpha}{2}}{- \quad {}^{(4-\alpha)}\pi^2 k^2 a \ \alpha \quad \frac{2-\alpha}{2} \quad {}_0^L C_n^2 \ z \quad z} \quad ^{\frac{1}{\alpha-2}},$$

$$a \ \alpha \quad - \quad {}^{(\alpha-4)}\pi^{-3/2} \frac{\frac{\alpha}{2}}{\frac{3-\alpha}{2}},$$

k α

ρ_0 r_0 C_n^2

$$k_0 \quad /L_0,$$

L_0

$\vec{v}$ $\vec{r}$

t_2 ϕ $\vec{r}, t_2$ $t_1 < t_2$ ϕ $\vec{r}, t_1$

$$\phi \ \vec{r}, t_2 \quad \phi \ \vec{r} \quad \vec{v} \ t_2 - t_1 \ , t_1 \ .$$

ρ_{0_i}

$\vec{v}_i$ Q $\rho_{0_{tot}}$

ρ_{0_i}

$$\rho_{0_{tot}}^{2-\alpha} \quad \sum_{i=1}^{Q} \rho_{0_i}^{2-\alpha}.$$

$$D_s \;\vec{p}, \vec{q}, t_1, t_2 \quad \mathcal{E} \quad s_{\hat{a}} \;\vec{p}, t_1 \; - s_{\hat{b}} \;\vec{q}, t_2 \quad ^2$$

$\mathcal{E}\{\cdots\}$ $s_{\hat{a}} \;\vec{p}, t_1$ t_1

$\vec{p}$ a $s_{\hat{b}} \;\vec{q}, t_2$

t_2 $\vec{q}$ b

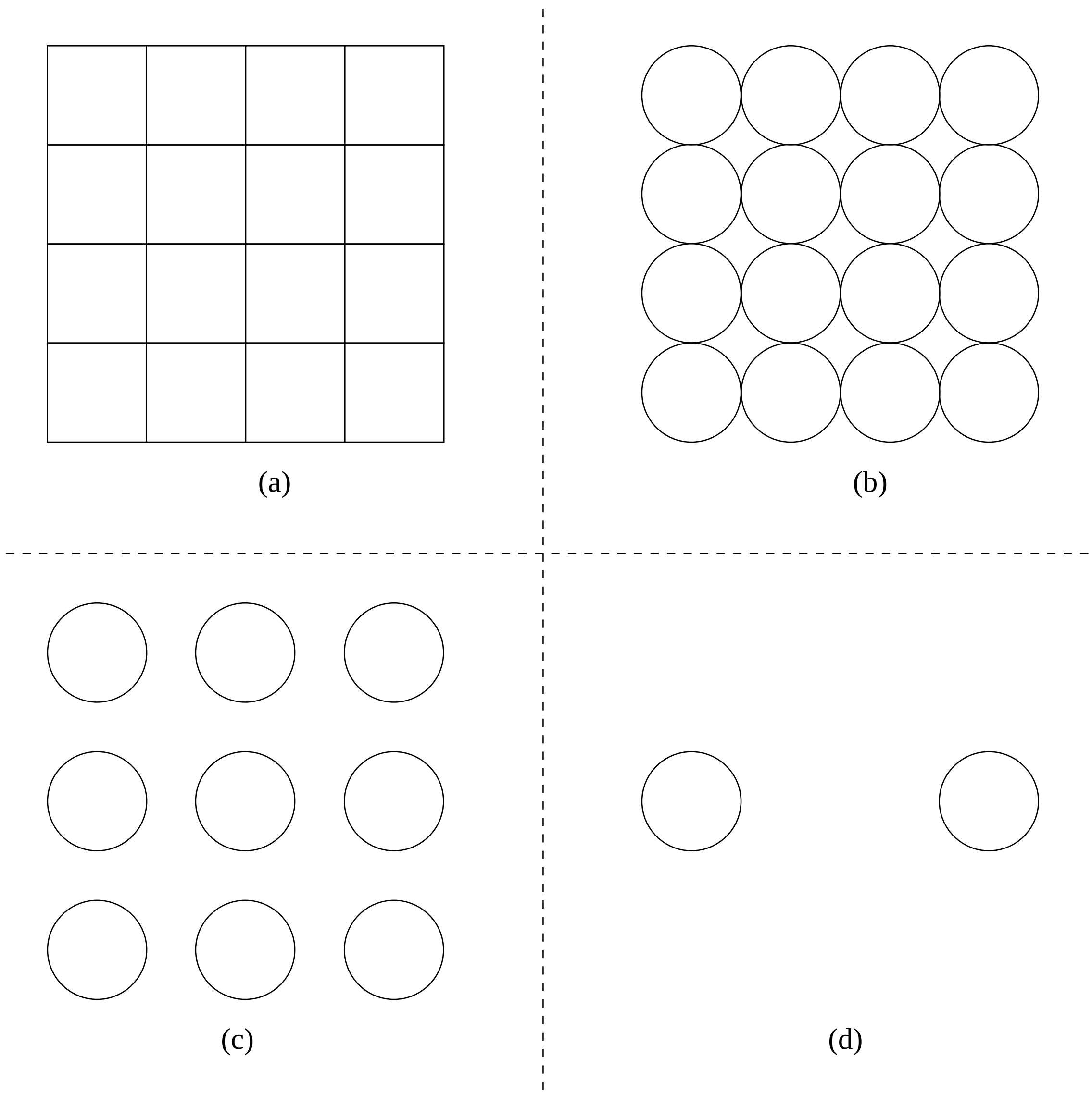
$$D_{s_{\hat{a}\hat{b}}} \;\vec{p}, \vec{q}, t_1, t_2 \quad \mathcal{E} \quad s^2_{\hat{a}} \;\vec{p}, t_1 \quad s^2_{\hat{b}} \;\vec{q}, t_2 \; - \; s_{\hat{a}} \;\vec{p}, t_1 \; s_{\hat{b}} \;\vec{q}, t_2$$

$$_{s_{\hat{a}\hat{b}}} \quad - \quad _{s_{\hat{a}\hat{b}}} \;\vec{q} - \vec{p} \quad \vec{v} \;t_2 - t_1 \quad ,$$

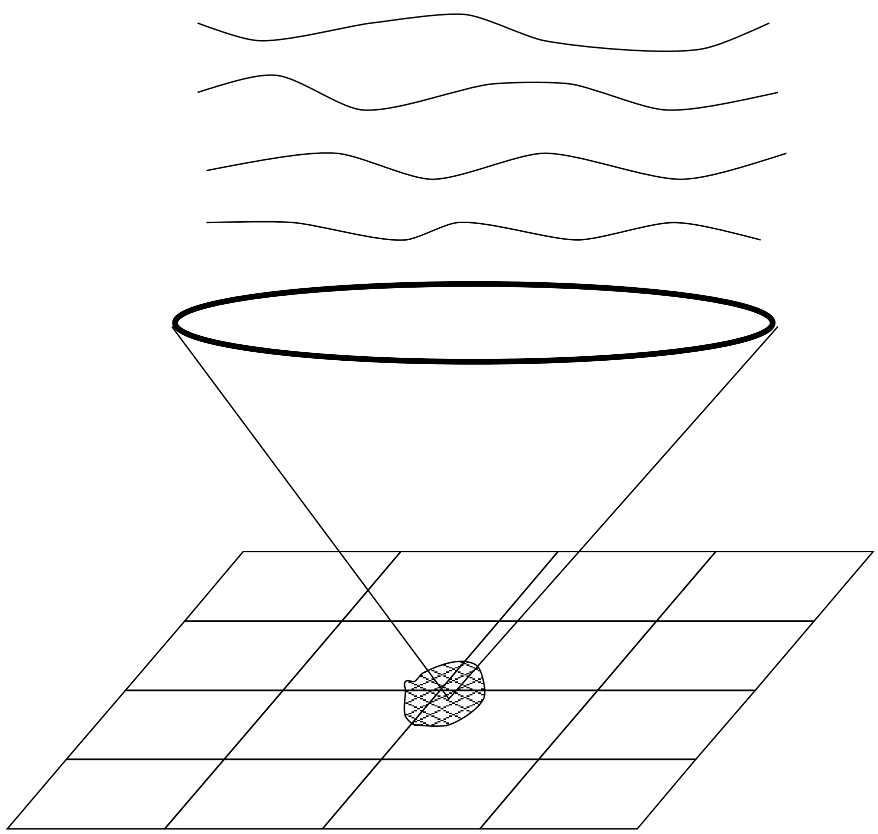
$\vec{v}$ $_s$

$$_{s_{\hat{a}\hat{b}}} \;\vec{q} - \vec{p} \quad \vec{v} \;t_2 - t_1 \quad \mathcal{E} \quad s_{\hat{a}} \;\vec{p}, t_1 \; s_{\hat{b}} \;\vec{q}, t_2 \quad .$$

a b *self slope structure function* a b *cross slope structure function*



(a)
(b)
(c)
(d)



a b

$$_s\ \vec{q} - \vec{p} \quad \vec{v}\ t_2 - t_1 \qquad \mathcal{E}\,\{s\ \vec{p}, t_1\ \ s\ \vec{q}, t_2\ \}\,.$$

$\vec{\rho}$ $\vec{p} - \vec{q}$

$\vec{\rho}$

M

$\vec{\rho}$ MN

$\vec{\rho}$

$$D_s\ \vec{\rho} \qquad \overline{NM} \sum_{m=1}^{M} \sum_{n=1}^{N}\ s\ \vec{q}_m, t_n\ \ - s\ \vec{p}_m, t_n\ \ ^2\,,$$

$D_s\ \vec{\rho}$ $\vec{\rho}$ $\vec{q}_m - \vec{p}_m$ m

$\vec{\rho}$ $\vec{q}_m - \vec{p}_m$ $s\ \vec{p}_m, t_n$

t_n $\vec{p}_m$

$_s$

$$SNR\ \ D_s\ \vec{\rho} \qquad \frac{\mathcal{E}\ \ D_s\ \vec{\rho}\ \ -}{\mathcal{E}\ \ \ D_s\ \vec{\rho}\ \ ^2\ \ - \mathcal{E}\ \ D_s\ \vec{\rho}\ \ ^2\ ^{1/2}},$$

Ds

1.4 Problem Description and Scope

-
-
- s
-
-
-
-

1.5 Organization

s

II. Slope Correlation Function and Slope Structure Function (SSF)

s

2.1 Zernike Polynomials

$$
\begin{aligned}
Z \quad _j\ r,\theta \quad & \sqrt{n \quad} R_n^m\ r\ \sqrt{\ } \quad m\theta \\
Z \quad _j\ r,\theta \quad & \sqrt{n \quad} R_n^m\ r\ \sqrt{\ } \quad m\theta \qquad m\ / \qquad , \\
Z_j\ r,\theta \quad & \sqrt{n \quad} R_n^0\ r \qquad\qquad m
\end{aligned}
$$

$$
R_n^m\ r \quad \sum_{s=0}^{(n-m)/2} \frac{-\ \ ^s\ n-s}{s\quad n\quad m\ /\ -s\quad n-m\ /\ -s} r^{n-2s}.
$$

j n m n m

$$
\begin{aligned}
& m \le n \\
& n - |m|
\end{aligned}
\quad .
$$

$$\int_{-\infty}^{\infty} r \; \theta W \; r \; Z_j \; r,\theta \; Z_{j'} \; r,\theta \qquad \delta_{jj'},$$

$$W \; r \qquad \begin{array}{ll} /\pi & r \leq \\ & r > \end{array} ,$$

$$\delta_{jj'} \qquad \begin{array}{ll} j & j' \\ & \end{array} .$$

$\phi \; r,\theta$

R

$$\phi \; R\rho,\theta \qquad \sum_j a_j Z_j \; \rho,\theta \; ,$$

$$\rho \quad r/R,$$

$$a_j \quad \overline{R^2} \int_{-\infty}^{\infty} r \; \theta W \; \frac{r}{R} \; \phi \; r,\theta \; Z_j \; \frac{r}{R},\theta \; .$$

$Q_j \; k,\phi$ $\qquad Z_j \; \rho,\theta$

$$W \; \rho \; Z_j \; \rho,\theta \qquad \int_{-\infty}^{\infty} {}^2\vec{k} \, Q_j \; k,\phi \qquad - \; \pi i\vec{k}\cdot\vec{\rho} \; ,$$

$i \quad \sqrt{-}$

$$\begin{array}{llll} Q \quad j \; k,\phi & & - \; {}^{(n-m)/2} i^m \sqrt{} & m\phi \\ Q \quad j \; k,\phi & \sqrt{n} \dfrac{J_{n+1} \; \pi k}{\pi} k & - \; {}^{(n-m)/2} i^m \sqrt{} & m\phi \\ Q_j \; k,\phi & & - \; {}^{n/2}, & m \end{array} .$$

j	m	n	$Z_j\ r,\theta$
			$r \quad \theta$
			$r \quad \theta$
			$\sqrt{\ } \ - \quad r^2$
			$\sqrt{\ }\, r^2 \quad \theta$
			$\sqrt{\ }\, r^2 \quad \theta$
			$\frac{3}{2} \ - \ r \quad r^3 \quad \theta$
			$\frac{3}{2} \ - \ r \quad r^3 \quad \theta$
			$\frac{3}{2}\, r^3 \quad \theta$
			$\frac{3}{2}\, r^3 \quad \theta$
			$\sqrt{\ } \ - \ r^2 \quad r^4$
			$\sqrt{\ } \ - \ r^2 \quad r^4 \quad \theta$
			$\sqrt{\ } \ - \ r^2 \quad r^4 \quad \theta$
			$\sqrt{\ }\, r^4 \quad \theta$
			$\sqrt{\ }\, r^4 \quad \theta$
			$\sqrt{\ } \quad r - \quad r^3 \quad r^5 \quad \theta$
			$\sqrt{\ } \quad r - \quad r^3 \quad r^5 \quad \theta$
			$\sqrt{\ } \ - \ r^3 \quad r^5 \quad \theta$
			$\sqrt{\ } \ - \ r^3 \quad r^5 \quad \theta$
			$\sqrt{\ }\, r^5 \quad \theta$
			$\sqrt{\ }\, r^5 \quad \theta$
			$\sqrt{\ } \ - \quad r^2 - \quad r^4 \quad r^6$
			$\sqrt{\ } \quad r^2 - \quad r^4 \quad r^6 \quad \theta$
			$\sqrt{\ } \quad r^2 - \quad r^4 \quad r^6 \quad \theta$
			$\sqrt{\ } \ - \ r^4 \quad r^6 \quad \theta$
			$\sqrt{\ } \ - \ r^4 \quad r^6 \quad \theta$
			$\sqrt{\ }\, r^6 \quad \theta$
			$\sqrt{\ }\, r^6 \quad \theta$
			$- \ r \quad r^3 - \quad r^5 \quad r^7 \quad \theta$
			$- \ r \quad r^3 - \quad r^5 \quad r^7 \quad \theta$
			$r^3 - \quad r^5 \quad r^7 \quad \theta$
			$r^3 - \quad r^5 \quad r^7 \quad \theta$
			$- \ r^5 \quad r^7 \quad \theta$
			$- \ r^5 \quad r^7 \quad \theta$
			$r^7 \quad \theta$

j

m n

2.2 Zernike Expansion Coefficient Spatial Covariance for Turbulence Induced Phase

D D R

xy

$\vec{u}$ $\vec{u}$ u

θ_0 $\vec{u}$ x

$$\phi \; R\vec{\rho} \quad \sum_{j=1}^{\infty} a_j Z_j \; \vec{\rho} \; ,$$

$$\phi \; R \; \vec{\rho}\,' \quad \vec{u} \quad \sum_{j'=1}^{\infty} a_{j'} \; \vec{u} \; Z_{j'} \; \vec{\rho}\,' \; .$$

ϕ ϕ

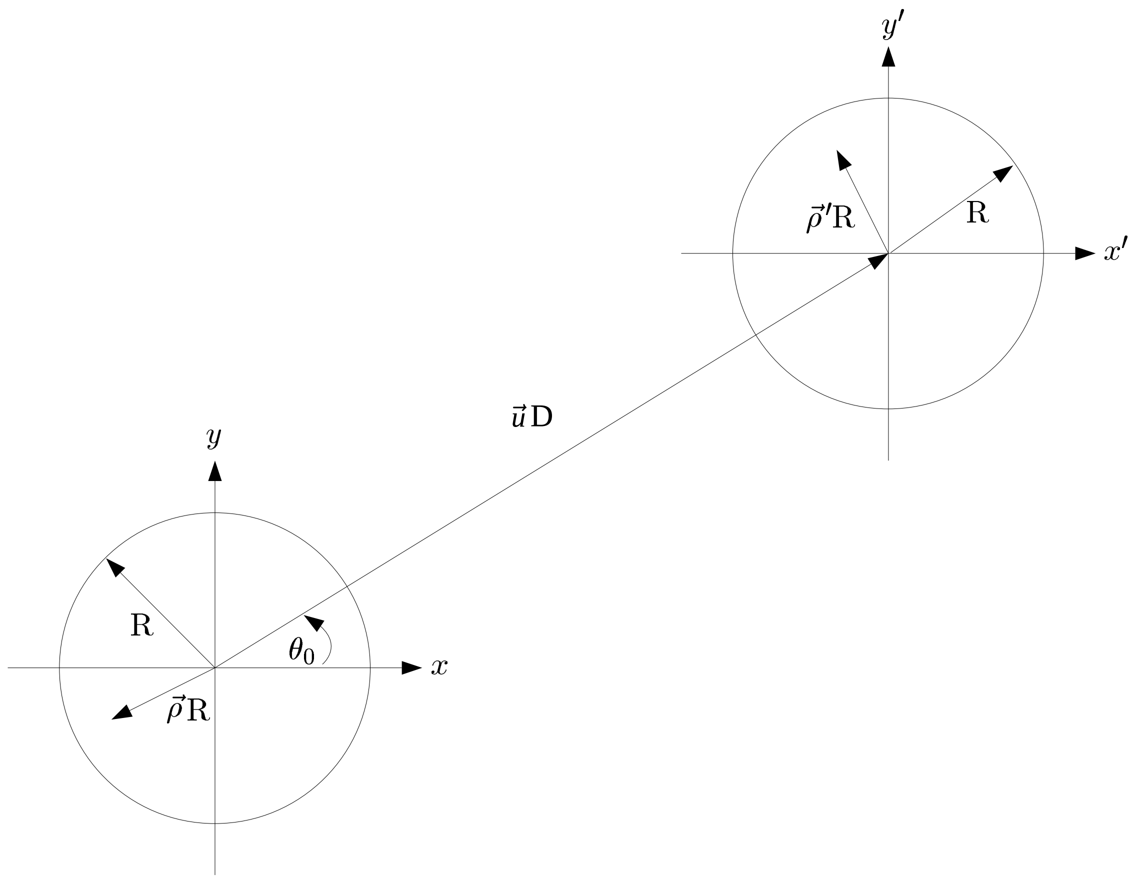
$$a_j \quad \int_{-\infty}^{\infty} \phi \; R\vec{\rho} \; Z_j \; \vec{\rho} \; W \; \vec{\rho} \quad \vec{\rho},$$

$$a_{j'} \; \vec{u} \quad \int_{-\infty}^{\infty} \phi \; R \; \vec{\rho}\,' \quad \vec{u} \quad Z_{j'} \; \vec{\rho}\,' \; W \; \vec{\rho}\,' \quad \vec{\rho}\,',$$

$W \; \vec{\rho}$

$$\mathcal{E} \quad a_j a_{j'}^* \; \vec{u} \quad \int_{-\infty}^{\infty} \vec{\rho} \; \int_{-\infty}^{\infty} \vec{\rho}\,' \mathcal{E} \quad \phi \; R\vec{\rho} \; \phi^* \; R \; \vec{\rho}\,' \quad \vec{u} \quad Z_j \; \vec{\rho} \; Z_{j'}^* \; \vec{\rho}\,' \; W \; \vec{\rho} \; W \; \vec{\rho}\,' \; ,$$

$\mathcal{E}\left\{\cdots\right\}$



R $D/$ xy

$\vec{u}D$

xy $x'y'$

$\vec{u}$ $|\vec{u}|$ D θ_0

$\vec{u}$ x

$$\mathcal{E} \quad a_j a_{j'}^* \ \vec{u} \qquad \overline{R^2} \ {}_0^{\infty} \ k \ {}_0^{2\pi} \ \theta \quad {}^{-i\,2\pi 2uk\cos(\theta-\theta_0)} \ {}_\phi \ k/R \ Q_j \ k,\theta \ Q_{j'}^* \ k,\theta \ ,$$

$Q_j \ k,\theta$

k θ

$C_n^2 \ z$

ρ_0 θ

$$\mathcal{E} \quad a_j a_{j'}^* \ \vec{u} \qquad c_1 \quad \frac{D}{\rho_0}^{(\alpha-2)} \quad \frac{\frac{\alpha}{2}}{- \quad \frac{2-\alpha}{2}} \quad n \qquad n' \qquad {}^{1/2} f_{jj'} \ u,\theta_0,k_0 \ ,$$

$$f_{jj'} \; u, \theta_0, k_0$$

$$m, n' \; / \quad j, j'$$
$$- \quad {}^{(n+n'-m+m')/2} \quad m \quad m' \; \theta_0 \; I_{m+m',n+1,n'+1} \quad u, k_0$$
$$- \quad {}^{(n+n'+2m+|m-m'|)/2} \quad m - m' \; \theta_0 \; I_{|m-m'|,n+1,n'+1} \quad u, k_0$$

$$m, n' \; / \quad j, j'$$
$$- \; - \quad {}^{(n+n'-m+m')/2} \quad m \quad m' \; \theta_0 \; I_{m+m',n+1,n'+1} \quad u, k_0$$
$$- \quad {}^{(n+n'+2m+|m-m'|)/2} \quad m - m' \; \theta_0 \; I_{|m-m'|,n+1,n'+1} \quad u, k_0$$

$$m, n' \; / \quad j \quad j'$$
$$- \quad {}^{(n+n'-m+m')/2} \quad m \quad m' \; \theta_0 \; I_{m+m',n+1,n'+1} \quad u, k_0$$
$$- \; - \quad {}^{(n+n'+2m+|m-m'|)/2} \quad m - m' \; \theta_0 \; I_{|m-m'|,n+1,n'+1} \quad u, k_0$$

$$m, n' \; / \quad j \quad j'$$
$$- \quad {}^{(n+n'-m+m')/2} \quad m \quad m' \quad \theta_0 \quad \pi \quad I_{m+m',n+1,n'+1} \quad u, k_0$$
$$- \; - \quad {}^{(n+n'+2m+|m-m'|)/2} \quad m - m' \quad \theta_0 \quad \pi \quad I_{|m-m'|,n+1,n'+1} \quad u, k_0 \quad ,$$

$$m' \quad j$$
$$- \quad {}^{(n+n'-m)/2}\sqrt{} \quad m\theta_0 \; I_{m,n+1,n'+1} \quad u, k_0$$

$$m \quad j'$$
$$- \quad {}^{(n+n'-m)/2}\sqrt{} \quad m \; \theta_0 \quad \pi \quad I_{m,n+1,n'+1} \quad u, k_0$$

$$m' \quad j$$
$$- \quad {}^{(n+n'-m)/2}\sqrt{} \quad m\theta_0 \; I_{m,n+1,n'+1} \quad u, k_0$$

$$m \quad j'$$
$$- \quad {}^{(n+n'-m)/2}\sqrt{} \quad m \; \theta_0 \quad \pi \quad I_{m,n+1,n'+1} \quad u, k_0$$

$$m \quad m'$$
$$- \quad {}^{(n+n')/2} I_{0,n+1,n'+1} \quad u, k_0$$

$$k_0 \quad \pi \frac{D}{L_0},$$

$$I_{\kappa,\mu,\nu} \; a, x_0 \quad {}_0^\infty \frac{x^{-1} J_\kappa \; ax \; J_\mu \; x \; J_\nu \; x}{x^2 \quad {x_0}^2 \;\; {}^{\alpha/2}} \; x,$$

c_1 D L_0 u

θ_0 $J_\kappa\ x$ $J_\mu\ x$ $J_\nu\ x$

κ μ ν [1]

j j'

$\mathcal{E}$ $a_j a_{j'}$

2.3 Slope Correlation Function and Slope Structure Function Evaluation

s

D_s

$$s_{\hat{a}}\ \vec{x}, t \quad \vec{r}\, W\ \vec{r} - \vec{x} \quad \nabla\phi\ \vec{r}, t\ \cdot a\ ,$$

$s_{\hat{a}}\ \vec{x}, t$ t

a $\vec{x}$ $W\ \vec{r}$

$\phi\ \vec{r}, t$ ∇

. $W\ \vec{r}$

$s\ \vec{x}, t$ $\vec{x}$

et. al.

[1] Evaluating Eqn. (2.21) is computational expensive and ultimately limits which problems can realistically be solved. Reference [15] presents a closed form solution for Eqn. (2.21), where $\vec{u} \geq 2$, consisting of infinite sums of hypergeometric functions of type ${}_4F_3$. They also state it is easier to numerically integrate Eqn. (2.21) than to use the closed form. This author has explored both approaches and agrees.

$Z_j \;\; \rho,\theta$

$W \;\; \rho$

$$\vec{\rho} W \;\; \rho \;\frac{\partial Z_j \;\; \rho,\theta}{\partial X} \qquad Z_j \quad ,\epsilon \;\; , \quad m \qquad , \qquad\quad , \qquad m \;\; /$$

X ϵ x

$$X \quad \rho \quad\; \theta - \epsilon \;\; .$$

$$_s \;\; \vec{x}_2 - \vec{x}_1 \qquad \mathcal{E}\,\{s \;\; \vec{x}_1 \;\; s \;\; \vec{x}_2 \;\; \}$$

$$\mathcal{E} \qquad \vec{\rho} W \;\; \rho \;\frac{\partial\phi \;\; R\vec{\rho}}{\partial X} \qquad \vec{\rho}\,' W \;\; \rho' \;\frac{\partial\phi \;\; R \;\; \vec{\rho}\,' \qquad \vec{u}}{\partial X'} \qquad ,$$

X X' $\leq \rho \leq \quad R$

$$\vec{u} \qquad \overline{R} \;\; \vec{x}_2 - \vec{x}_1 \;\; .$$

$$_s \;\; \vec{x}_2 - \vec{x}_1 \qquad \mathcal{E} \qquad \vec{\rho} W \;\; \rho \;\frac{\partial \overset{\infty}{\underset{j=1}{}} \; a_j Z_j \;\; \vec{\rho}}{\partial X} \qquad \vec{\rho}\,' W \;\; \rho' \;\frac{\partial \overset{\infty}{\underset{j'=1}{}} \; a_{j'} \;\; \vec{u} \;\; Z_{j'} \;\; \vec{\rho}\,'}{\partial X'} \qquad .$$

$$_s \;\; \vec{x}_2 - \vec{x}_1 \qquad \mathcal{E} \qquad \underset{j,m=1}{} \; a_j Z_j \quad ,\epsilon \qquad \underset{j',m'=1}{} \; a_{j'} \;\; \vec{u} \;\; Z_{j'} \quad ,\epsilon$$

$$\underset{j,m=1}{}\;\underset{j',m'=1}{} \;\; \mathcal{E} \;\; a_j a_{j'} \;\; \vec{u} \quad Z_j \quad ,\epsilon \;\; Z_{j'} \quad ,\epsilon \;\; .$$

x y

ϵ ϵ $\pi/$ Z_j ,

Z_j $,\pi/$ m $j \leq$

Z_j j j'

both even *both odd*

$$\mathcal{E} \quad a_j a^*_{j'} \; \vec{u} \qquad c_1 \quad \frac{D}{\rho_0} \quad ^{(\alpha-2)} \quad \frac{\frac{\alpha}{2}}{- \quad \frac{2-\alpha}{2}} \quad n \qquad n' \qquad ^{1/2} f_{jj'} \; u, \theta_0, k_0 \; ,$$

$$f_{jj'} \; u, \theta_0, k_0$$

$$- \quad ^{(n+n')/2} \qquad \theta_0 \; I_{2,n+1,n'+1} \quad u, k_0 \qquad j, j'$$

$$- \quad ^{(n+n'+2)/2} I_{0,n+1,n'+1} \quad u, k_0$$

$$- \; - \quad ^{(n+n')/2} \qquad \theta_0 \; I_{2,n+1,n'+1} \quad u, k_0 \qquad j, j' \qquad ,$$

$$- \quad ^{(n+n'+2)/2} I_{0,n+1,n'+1} \quad u, k_0$$

$$_s \; \vec{x}_2 - \vec{x}_1 \qquad _s \; \vec{x}_1 - \vec{x}_2 \; .$$

$_s$

$_s$ D_s

	j	m	n	Z_j ,	Z_j , $\frac{\pi}{2}$
					$\frac{3}{2}$
				$\frac{3}{2}$	
				$\sqrt{\ }$	
					$\sqrt{\ }$
				$\sqrt{\ }$	
					$\sqrt{\ }$
					$\sqrt{\ }$
				$\sqrt{\ }$	
				$\sqrt{\ }$	
					$\sqrt{\ }$
					$\frac{5}{2}$
				$\frac{5}{2}$	
					$\sqrt{\ }$
				$\sqrt{\ }$	
				$\sqrt{\ }$	
					$\sqrt{\ }$
					$\sqrt{\ }$
				$\sqrt{\ }$	
				$\sqrt{\ }$	
					$\sqrt{\ }$
					$\sqrt{\ }$
				$\sqrt{\ }$	
				$\sqrt{\ }$	
					$\sqrt{\ }$

Z_j $, \epsilon$ $j \leq$ m

s

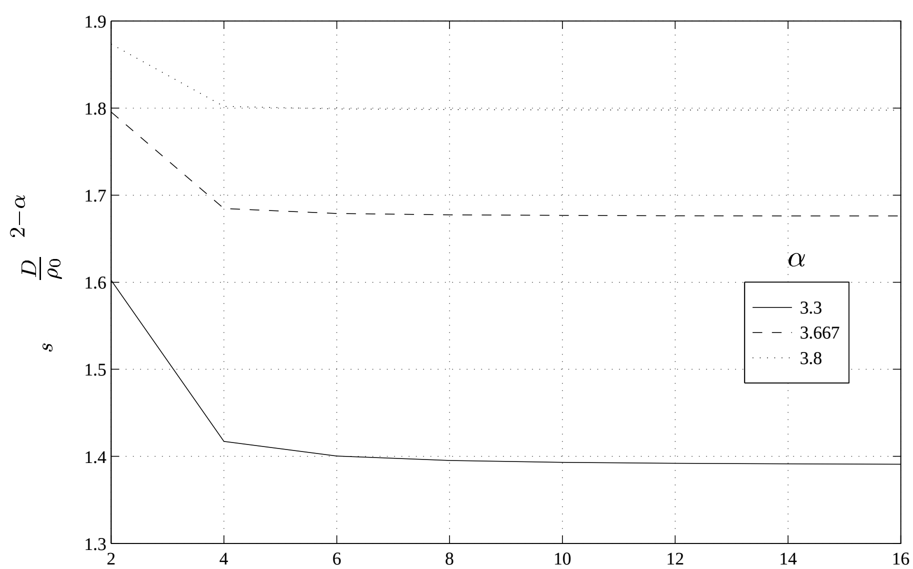
D_s

s $\vec{u}$

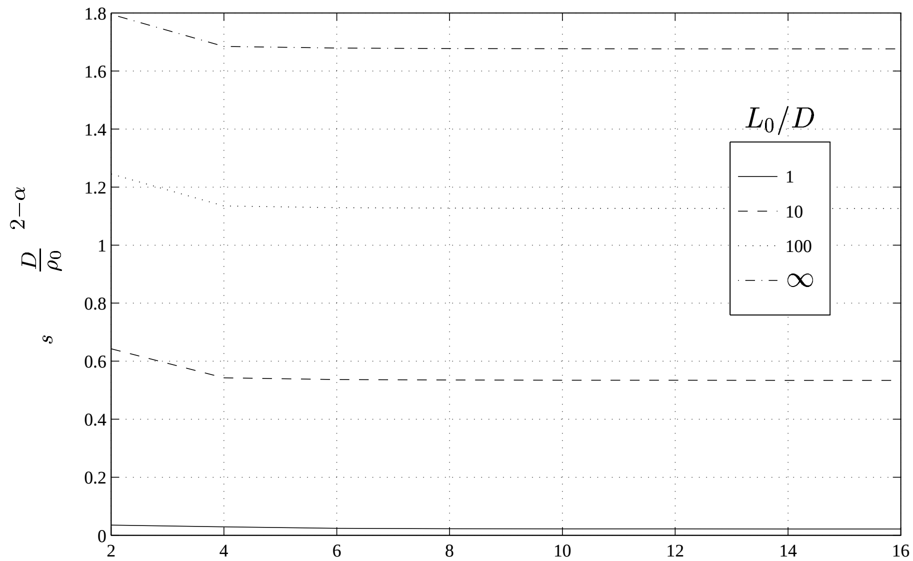
s

s

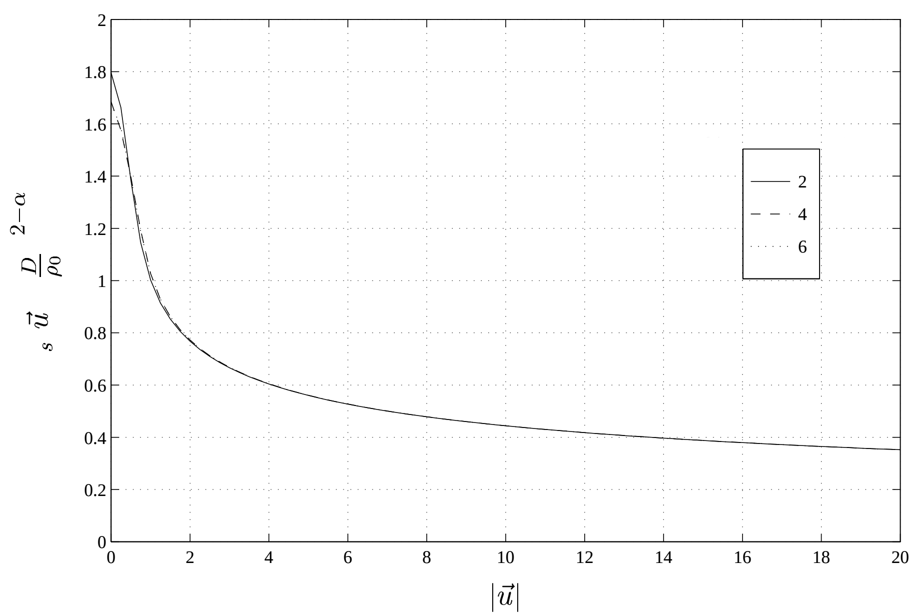
α



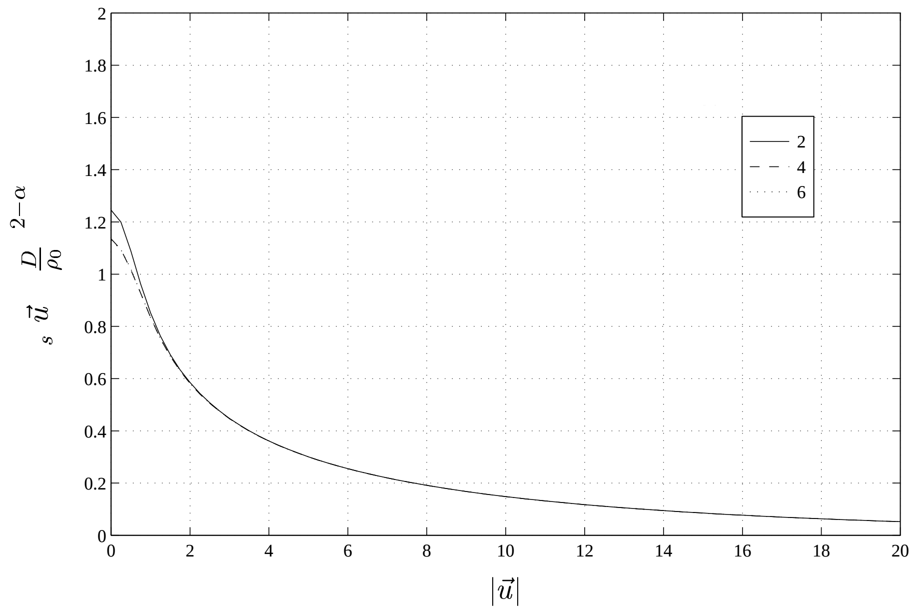
s $\frac{D}{\rho_0}^{2-\alpha}$ L_0/D ∞ α



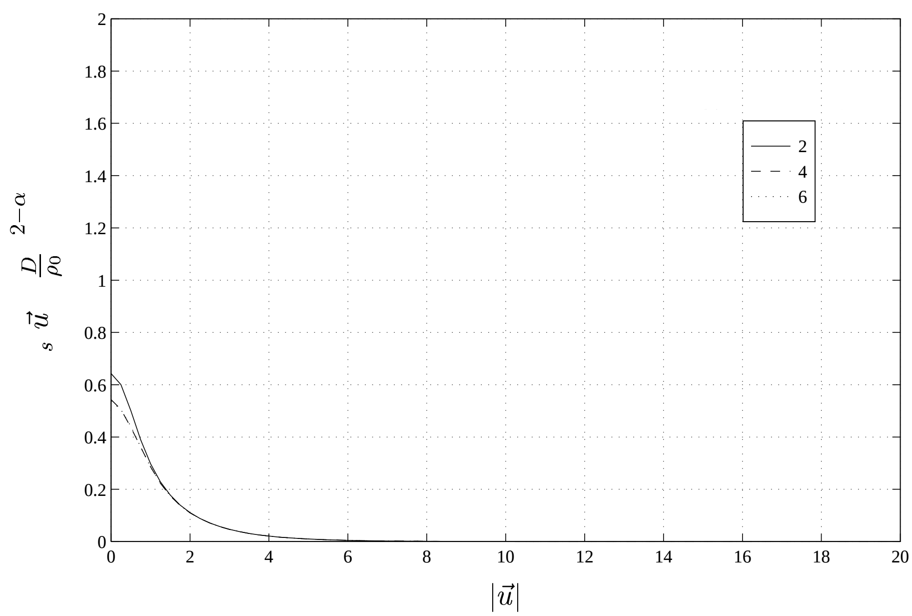
s $\frac{D}{\rho_0}$ $^{2-\alpha}$ α / L_0/D



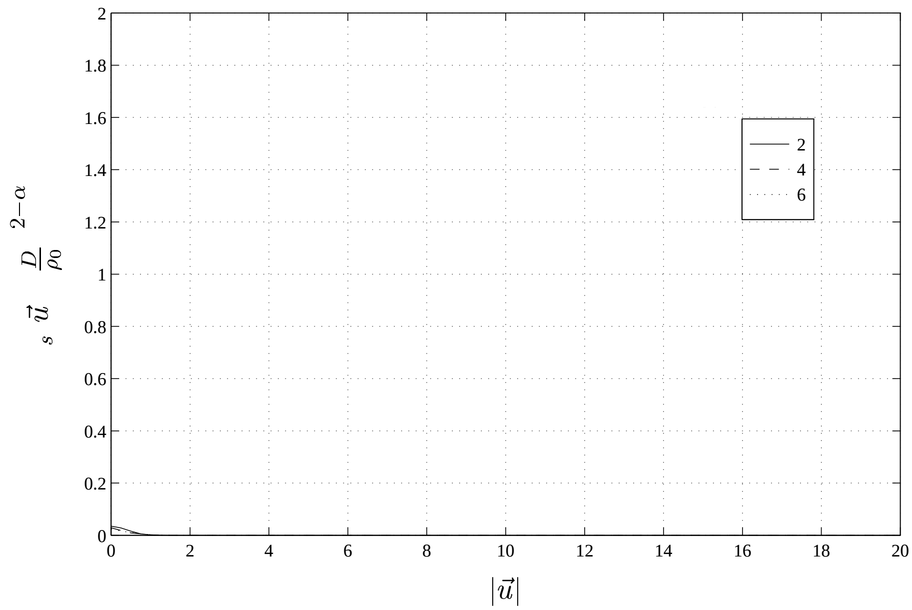
s $\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$ ∞ θ_0



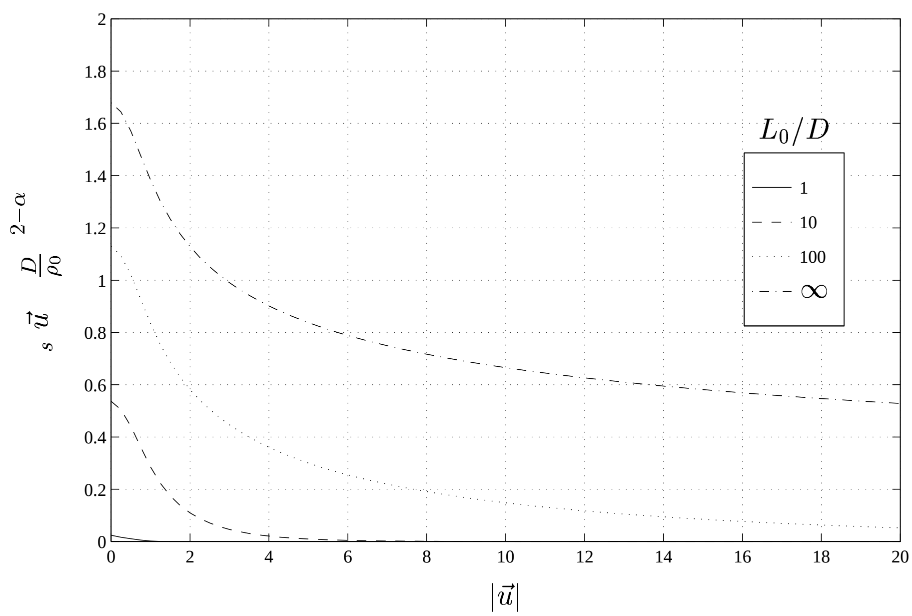
$_s\ \vec{u}\ \ \frac{D}{\rho_0}\ ^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



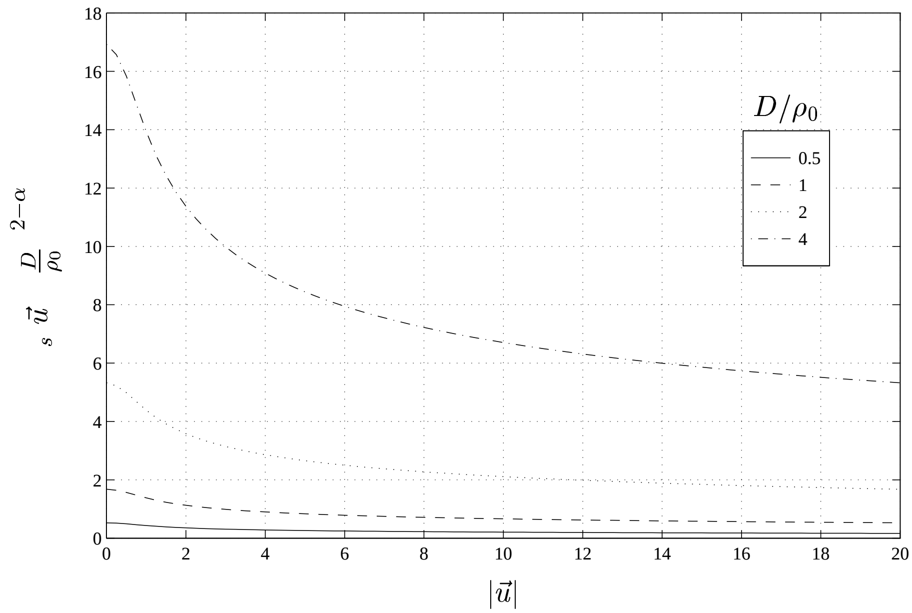
$_s\ \vec{u}\ \ \frac{D}{\rho_0}\ ^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



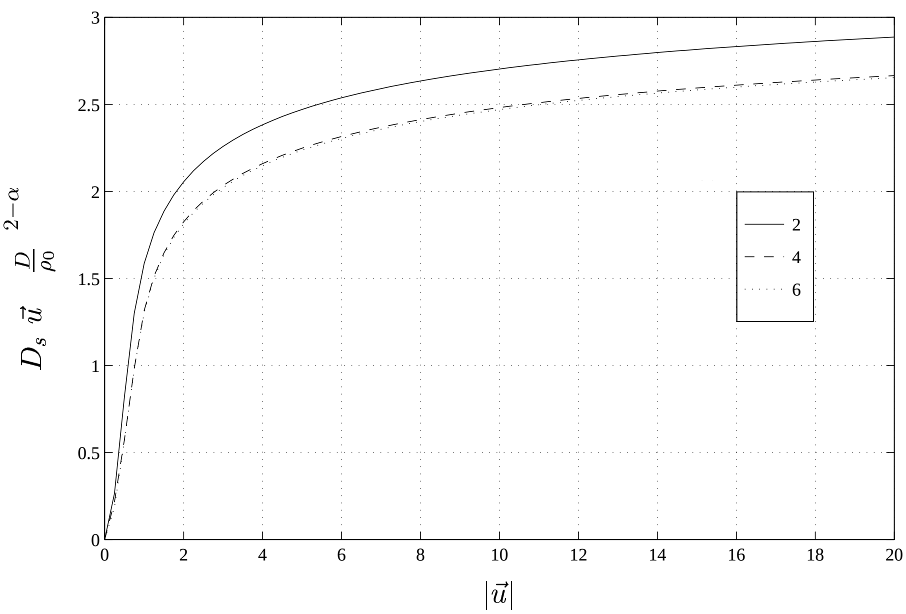
$_s\,\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



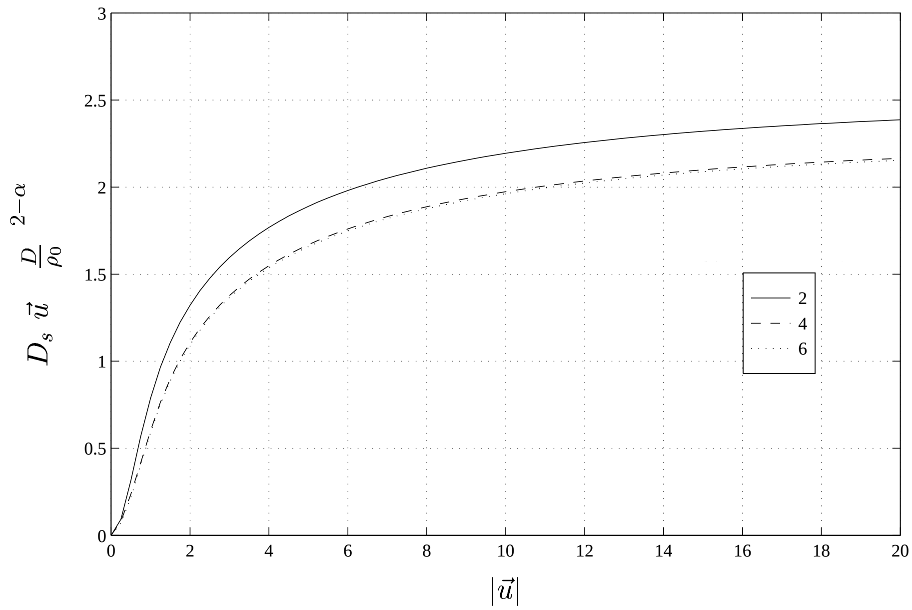
$_s\,\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|u|$ L_0/D α . θ_0



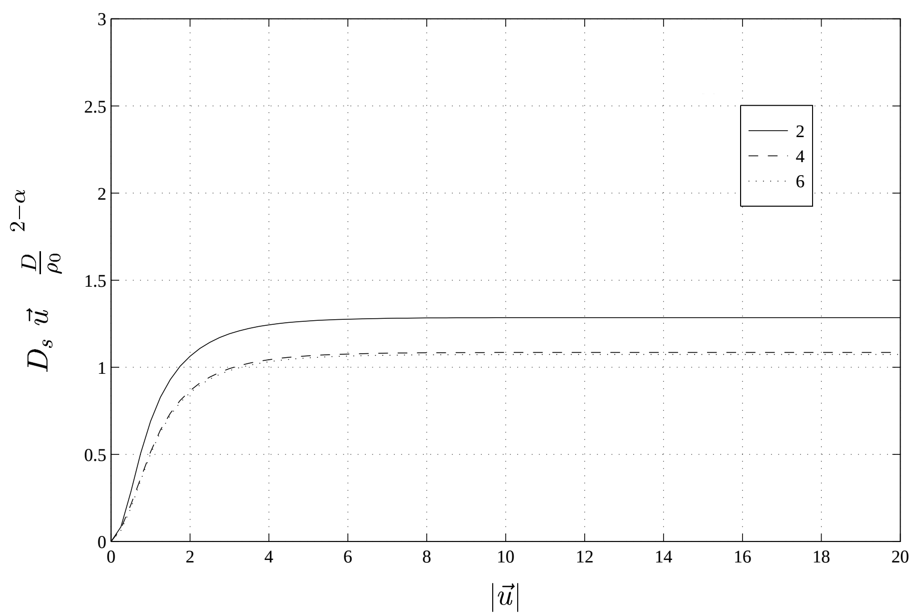
$_s\ \vec{u}$ $|\vec{u}|$ $\frac{D}{\rho_0}$ α .
$\frac{L_0}{D}$ ∞ θ_0



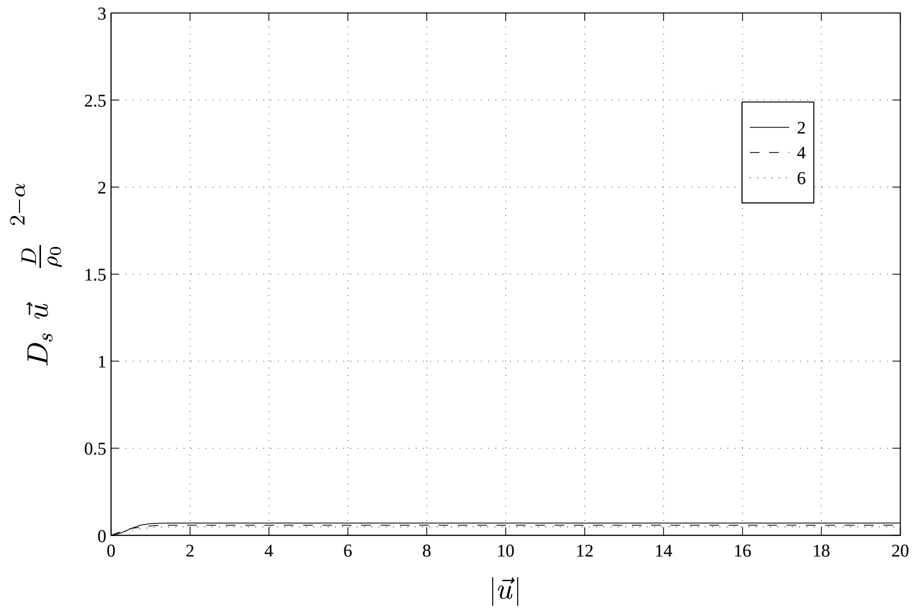
$D_s\ \vec{u}\ \frac{D}{\rho_0}\ ^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$ ∞ θ_0



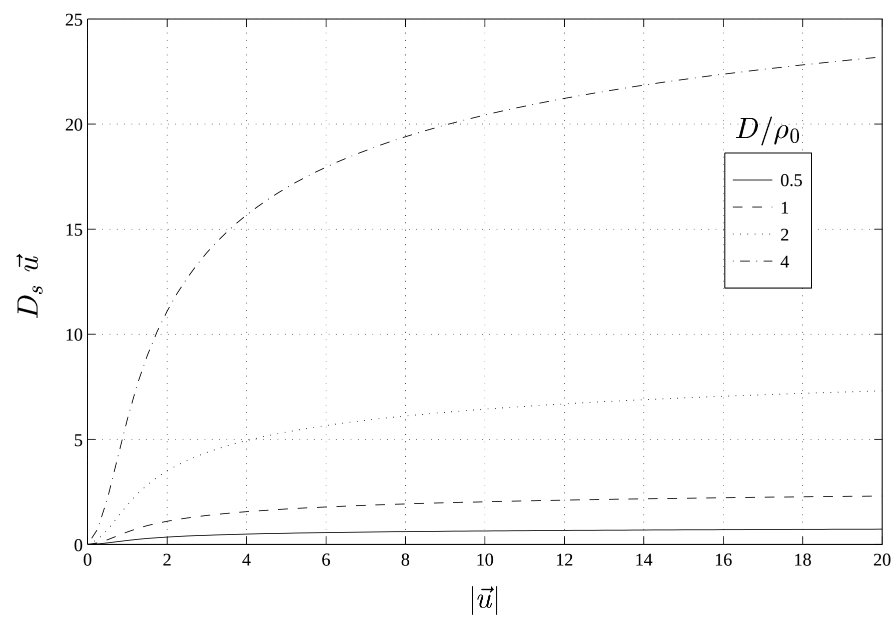
D_s $\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



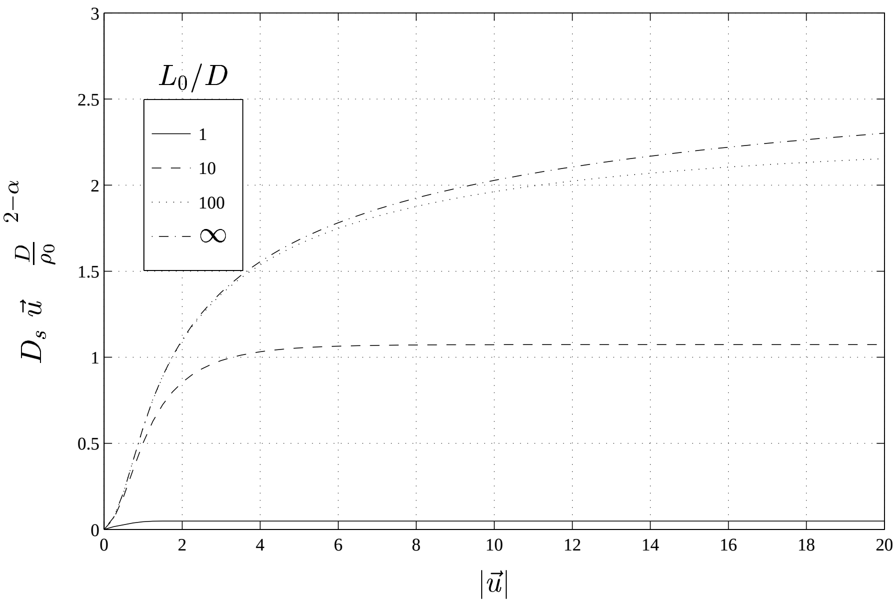
D_s $\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



D_s $\vec{u}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ $|\vec{u}|$ α .
$\frac{L_0}{D}$. θ_0



D_s $\vec{u}$ $|\vec{u}|$ D/ρ_0 α . $\frac{L_0}{D}$ ∞ θ_0



$D_s\ \vec{u}\quad \frac{D}{\rho_0}\ ^{2-\alpha}\qquad |u|\qquad L_0/D\qquad \alpha\qquad .\qquad \theta_0$

III. SSF Estimator Signal-to-Noise Ratio

3.1 H-WFS Specification

$$W \ \vec{r} \qquad \begin{matrix} /\pi & r \leq \\ & r > \end{matrix} \ ,$$

$r \quad |\vec{r}|$

D

i	$\vec{x}_i \ \frac{x}{D}, \frac{y}{D}$	i	$\vec{x}_i \ \frac{x}{D}, \frac{y}{D}$

D

3.2 Slope Measurement Model

$s_{\hat{a}}\ \vec{\rho}, t$

$s_{\hat{a}}\ \vec{p}, t$ $n\ \vec{p}, t$

$$s_{\hat{a}}\ \vec{p}, t \quad s_{\hat{a}}\ \vec{p}, t \quad n\ \vec{p}, t\ .$$

-
-
- σ_n^2
-

3.3 Slope Measurement Correlation

$$\mathcal{E}\left\{s\ \vec{p}, t_1\ s\ \vec{q}, t_2\ \right\}.$$

$\vec{q} - \vec{p}$

$\vec{v}\ t_2 - t_1$ $\vec{v}$

$$\begin{aligned}
&\mathcal{E}\left\{s\ \vec{p}, t_1\ s\ \vec{q}, t_2\ \right\} \\
&\quad \mathcal{E}\left\{\ s\ \vec{p}, t_1 \quad n\ \vec{p}, t_1 \quad s\ \vec{q}, t_2 \quad n\ \vec{q}, t_2\ \right\} \\
&\quad \mathcal{E}\left\{s\ \vec{p}, t_1\ s\ \vec{q}, t_2\ \right\} \quad \mathcal{E}\left\{s\ \vec{p}, t_1\ n\ \vec{q}, t_2\ \right\} \\
&\quad \mathcal{E}\left\{s\ \vec{q}, t_2\ n\ \vec{p}, t_1\ \right\} \quad \mathcal{E}\left\{n\ \vec{p}, t_1\ n\ \vec{q}, t_2\ \right\}
\end{aligned}$$

$_s$ σ_n^2 $\vec{p}$ $\vec{q}$ t_1 t_2

$_s$ $\vec{q} - \vec{p}$ $\vec{v}$ $t_2 - t_1$

$_s$

3.4 SSF Estimate SNR Definition

D_s $\vec{\rho}$

$$SNR \quad D_s \; \vec{\rho} \qquad \frac{\mathcal{E} \quad D_s \; \vec{\rho} \quad -}{\mathcal{E} \quad D_s \; \vec{\rho} \quad ^2 \quad - \mathcal{E} \quad D_s \; \vec{\rho} \quad ^2 \; ^{1/2}}.$$

D_s $\vec{\rho}$

3.5 First Moment of the SSF Estimator

$$\begin{aligned}
\mathcal{E} \quad D_s \; \vec{\rho} \qquad & \mathcal{E} \quad \overline{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} \quad s \; \vec{q}, t_n \quad - s \; \vec{p}, t_n \quad ^2 \\
& \overline{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} \mathcal{E} \quad s \; \vec{q}, t_n \quad - s \; \vec{p}, t_n \quad ^2 \\
& \overline{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} \mathcal{E} \quad s \; \vec{q}, t_n \quad n \; \vec{q}, t_n \quad - \; s \; \vec{p}, t_n \quad n \; \vec{p}, t_n \quad ^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{E} \quad D_s \; \vec{\rho} \qquad & \overline{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} \sigma_n^2 \quad _s \; \vec{} \; - \quad _s \; \vec{q} - \vec{p} \\
& \sigma_n^2 \quad _s \; \vec{} \; - \quad _s \; \vec{\rho} \; .
\end{aligned}$$

σ_n^2

3.6 Second Moment of SSF Estimator

$$\mathcal{E} \quad D_s \; \vec{\rho} \;^2 \qquad \mathcal{E} \quad \frac{}{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} s \; \vec{q}, t_n \; - s \; \vec{p}, t_n \;^2 \;^2 \quad .$$

$$\sum_{a=1}^{A} \sum_{b=1}^{B} f \; a, b \;^2 \qquad \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{c=1}^{C} \sum_{d=1}^{D} f \; a, b \; f \; c, d$$

$$\mathcal{E} \quad D_s \; \vec{\rho} \;^2$$

$$\mathcal{E} \quad \frac{}{NM \;^2} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{\mu=1}^{M} \sum_{\nu=1}^{N} s \; \vec{q}_m, t_n \; - s \; \vec{p}_m, t_n \;^2 \; s \; \vec{q}_\mu, t_\nu \; - s \; \vec{p}_\mu, t_\nu \;^2 \quad .$$

$$\mathcal{E} \quad D_s \; \vec{\rho} \;^2 \qquad \frac{}{NM \;^2} \sum_{n=1}^{N} \sum_{\nu=1}^{N} \sum_{m=1}^{M} \sum_{\mu=1}^{M} \mathcal{E} \quad s \; \vec{q}_m, t_n \; - s \; \vec{p}_m, t_n \;^2 \; s \; \vec{q}_\mu, t_\nu \; - s \; \vec{p}_\mu, t_\nu \;^2 \quad .$$

3.6.1 Expectation Simplification.

$$
\begin{array}{lll}
\vec{x}_1 & \vec{p}_m & \vec{v}t_n \\
\vec{x}_2 & \vec{q}_m & \vec{v}t_n \\
\vec{x}_3 & \vec{p}_\mu & \vec{v}t_\nu \\
\vec{x}_4 & \vec{q}_\mu & \vec{v}t_\nu
\end{array} .
$$

$\vec{x}_1$ $\vec{x}_2$

subaperture pair $\vec{x}_3$ $\vec{x}_4$

set

$$\mathcal{E} \quad s \ \vec{x}_2 \ - s \ \vec{x}_1 \ ^2 \ s \ \vec{x}_4 \ - s \ \vec{x}_3 \ ^2 \ .$$

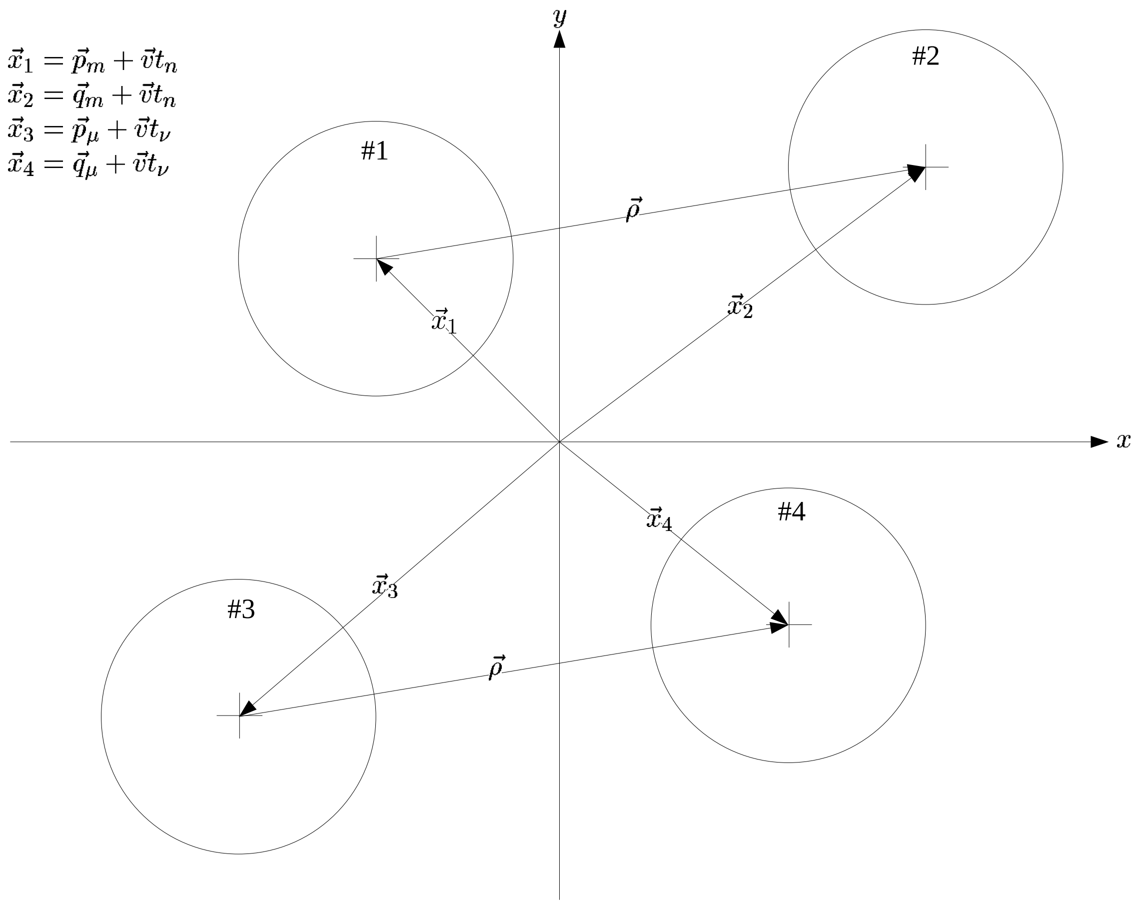
$$\mathcal{E} \quad s \ \vec{x}_2 \quad n \ \vec{x}_2 \ - \ s \ \vec{x}_1 \quad n \ \vec{x}_1 \ ^2 \ s \ \vec{x}_4 \quad n \ \vec{x}_4 \ - \ s \ \vec{x}_3 \quad n \ \vec{x}_3 \ ^2 \ .$$

u_1 u_2 u_3 u_4

$$\mathcal{E}\left\{u_1u_2u_3u_4\right\} \quad \mathcal{E}\left\{u_1u_2\right\}\mathcal{E}\left\{u_3u_4\right\} \quad \mathcal{E}\left\{u_1u_3\right\}\mathcal{E}\left\{u_2u_4\right\} \quad \mathcal{E}\left\{u_1u_4\right\}\mathcal{E}\left\{u_2u_3\right\}.$$

s

σ_n^2



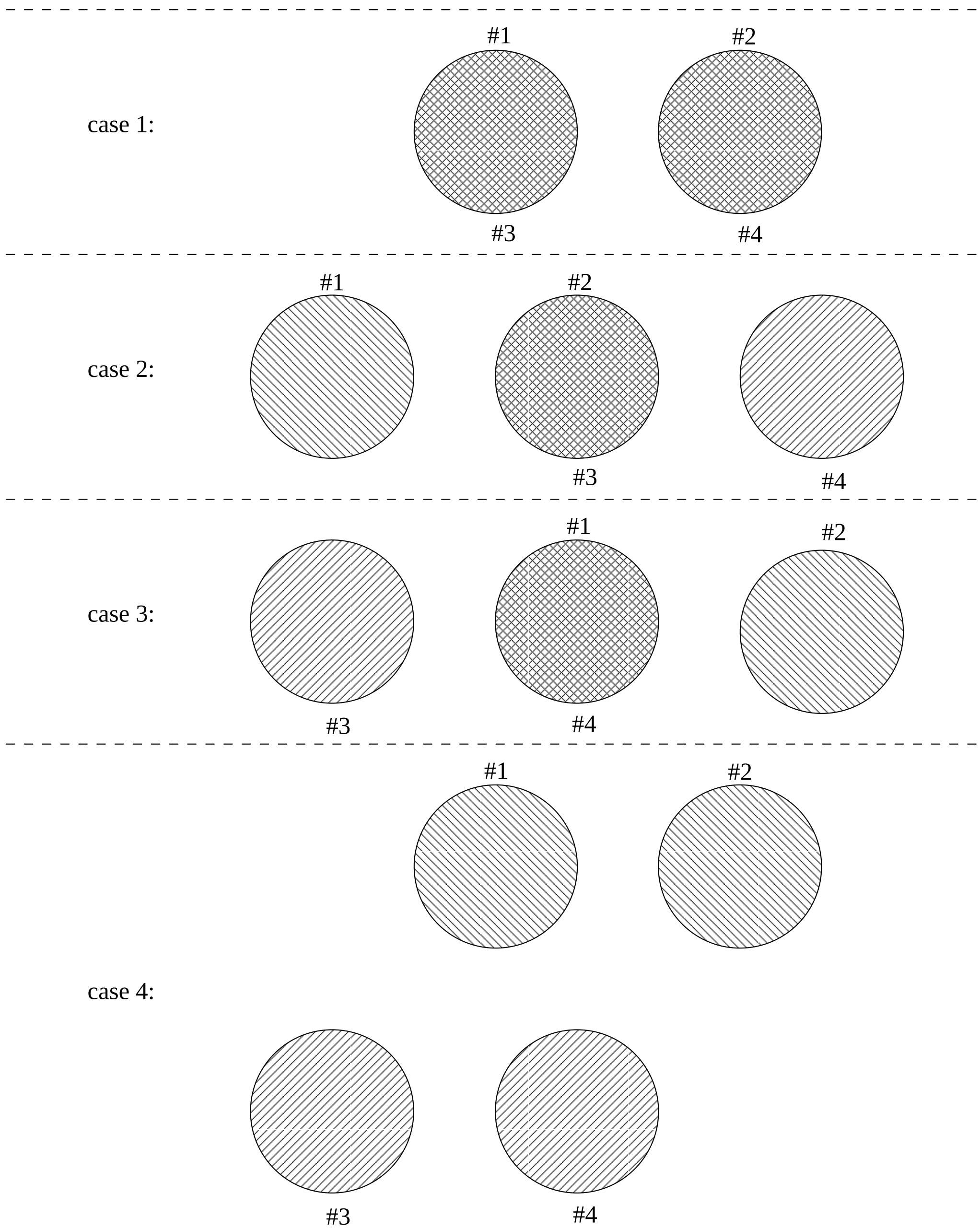
$\vec{\rho}$

$\vec{x}_1$ $\vec{x}_2$ $\vec{x}_3$ $\vec{x}_4$ xy

pair $\vec{\rho}$ *pair*

xy

a set of subaperture pairs



or

s σ_n^2

$$\vec{v}\ t_\nu - t_n \quad \vec{v}\tau\ \nu - n\ ,$$

τ ν n

$$\mathcal{E} \quad s\ \vec{q}_m, t_n\ - s\ \vec{p}_m, t_n\ ^2\ s\ \vec{q}_\mu, t_\nu\ - s\ \vec{p}_\mu, t_\nu\ ^2 \quad F_{sm}\ \vec{\rho}, \sigma_n, \vec{v}\tau\ \nu - n\ , \vec{p}_m, \vec{q}_m, \vec{p}_\mu, \vec{q}_\mu$$

$$F_{sm}\ \vec{\rho}, \sigma_n, \vec{v}\tau\ \nu - n\ , \vec{p}_m, \vec{q}_m, \vec{p}_\mu, \vec{q}_\mu$$

$$\sigma_n^4 \quad \sigma_n^2 \ {}_s \quad {}^2_s \quad - \quad \sigma_n^2 \ {}_s\ \vec{\rho} \qquad \vec{p}_m \quad \vec{p}_\mu$$

$$- \quad {}_s \quad {}_s\ \vec{\rho} \quad {}^2_s\ \vec{\rho} \qquad \vec{q}_m \quad \vec{q}_\mu \quad \nu - n$$

$$\sigma_n^4 \quad \sigma_n^2 \ {}_s \quad {}^2_s \quad - \quad \sigma_n^2 \ {}_s\ \vec{\rho}$$

$$- \quad {}_s \quad {}_s\ \vec{\rho} \quad {}^2_s\ \vec{\rho} \quad \sigma_n^2 \ {}_s \quad \vec{\rho} \qquad \vec{q}_m \quad \vec{p}_\mu$$

$${}_s \quad {}_s \quad \vec{\rho} - \quad {}_s\ \rho \quad {}_s \quad \vec{\rho} \qquad \vec{q}_\mu \quad \vec{p}_m \quad \nu - n$$

$${}^2_s \quad \vec{\rho}$$

.

$$\sigma_n^4 \quad \sigma_n^2 \ {}_s \quad {}^2_s \quad - \quad \sigma_n^2 \ {}_s\ \vec{\rho}$$

$$- \quad {}_s \quad {}_s\ \vec{\rho} \quad {}^2_s\ \vec{\rho} \quad {}^2_s\ \vec{p}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n$$

$$- \quad {}_s\ \vec{p}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n \quad {}_s\ \vec{p}_\mu - \vec{q}_m \quad \vec{v}\tau\ \nu - n$$

$${}^2_s\ \vec{p}_\mu - \vec{q}_m \quad \vec{v}\tau\ \nu - n$$

$$- \quad {}_s\ \vec{p}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n \quad {}_s\ \vec{q}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n$$

$${}_s\ \vec{p}_\mu - \vec{q}_m \quad \vec{v}\tau\ \nu - n \quad {}_s\ \vec{q}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n$$

$${}^2_s\ \vec{q}_\mu - \vec{p}_m \quad \vec{v}\tau\ \nu - n$$

3.6.2 Four Dimensional Summation Simplification.

$\nu - n$

$$\sum_{a=1}^{N}\sum_{b=1}^{N} f\ a - b \qquad N \sum_{c=-N}^{N} \quad - \frac{|c|}{N} \quad f\ c$$

$f\ a - b \qquad f\ b - a$

$$\sum_{a=1}^{N}\sum_{b=1}^{N} f\ a - b \qquad - \delta_{0,c}\ N \sum_{c=0}^{N} \quad - \frac{|c|}{N} \quad f\ c\ ,$$

c

$\delta_{0,c}$.

s

m μ

$\vec{\rho}$

$\vec{x}_1$ $\vec{x}_2$ $\vec{x}_3$ $\vec{x}_4$ *set of subaperture pairs*

$$\mathcal{E} \quad D_s \; \vec{\rho} \quad ^2 \qquad \overline{NM^2} \sum_{i=0}^{N} \; - \delta_{0,i} \qquad - \frac{|i|}{N} \sum_{j=1}^{M_r} w \; j \; F_{sm} \; \vec{\rho}, \sigma_n, \vec{v}\tau i, \vec{p}_m^{(j)}, \vec{q}_m^{(j)}, \vec{p}_\mu^{(j)}, \vec{q}_\mu^{(j)} \; ,$$

F_{sm} $w \; j$

M_r

$\vec{\rho}$ $M_r <$ M^2

j j $w \; j$

$w \; j$

$\vec{p}_\mu^{(j)}$ $\vec{p}_m^{(j)}$ $\vec{q}_\mu^{(j)}$ $\vec{q}_m^{(j)}$ i

$$\sum_{j=1}^{M_r} w \; j \qquad M^2.$$

m μ

j

3.7 Final SSF Estimator SNR Expression

s

ρ_0 L_0/D α N

3.8 Extension to a Multi-layer Atmospheric Model

ϕ_{tot} $\vec{r}, t$ Q

$$\phi_{tot} \ \vec{r}, t \quad \sum_{l=1}^{Q} \phi_l \ \vec{r}, t \ ,$$

ϕ_l $\vec{r}, t$ l

$$s_{\hat{a}_{tot}} \ \vec{x}, t \quad \vec{r} \, W \ \vec{r} - \vec{x} \quad \phi_{tot} \ \vec{r}, t \ \cdot a$$

$$\vec{r} \, W \ \vec{r} - \vec{x} \quad \sum_{l=1}^{Q} \phi_l \ \vec{r}, t \quad \cdot a$$

	M	M^2	M_r
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
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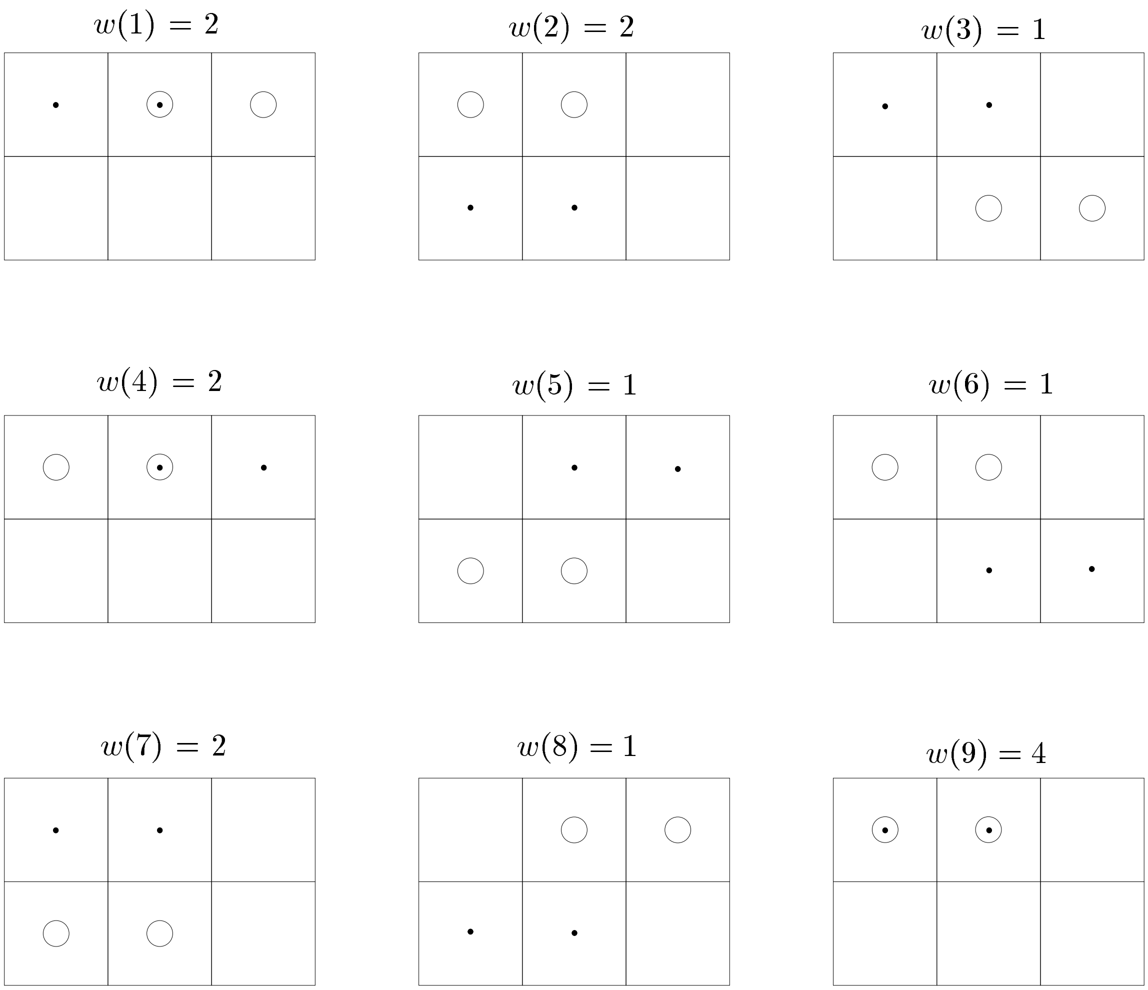
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			
$\times$			

$\times$
$\times$
D
$\vec{\rho}$ $D,$ M
$\vec{\rho}$ M^2

M_r
$\vec{\rho}$ j
w j



F_{sm} $\vec{\rho}, \sigma_n, \vec{v}\tau i, \vec{p}_m^{(j)}, \vec{q}_m^{(j)}, \vec{p}_\mu^{(j)}, \vec{q}_\mu^{(j)}$

$\vec{\rho}$ $D,$ $\circ$

n

ν

M_r

w j

$$\prod_{l=1}^{Q} \quad \vec{r}\, W \; \vec{r}-\vec{x} \quad \phi_l \; \vec{r},t \;\; \cdot a$$

$$\prod_{l=1}^{Q} s_{\hat{a}_l(\vec{x},t)}.$$

$s_{\hat{a}_{tot}}$

s

s_{tot}

$$_{s_{tot}} \; \vec{x}_2-\vec{x}_1 \qquad \mathcal{E}\left\{s_{tot} \; \vec{x}_1 \;\; s_{tot} \; \vec{x}_2 \;\right\}.$$

$$\begin{aligned}
_{s_{tot}} \; \vec{x}_2-\vec{x}_1 \qquad & \mathcal{E}\left\{s_{tot} \; \vec{x}_1 \;\; s_{tot} \; \vec{x}_2 \;\right\} \\
& \mathcal{E} \quad \prod_{l=1}^{Q} s_{\hat{a}_l} \; \vec{x}_1 \quad \prod_{l'=1}^{Q} s_{\hat{a}_{l'}} \; \vec{x}_2 \\
& \prod_{l=1}^{Q} \prod_{l'=1}^{Q} \mathcal{E} \quad s_{\hat{a}_l} \; \vec{x}_1 \;\; s_{\hat{a}_{l'}} \; \vec{x}_2 \\
& \prod_{l=1}^{Q} \mathcal{E}\left\{s_{\hat{a}_l} \; \vec{x}_1 \;\; s_{\hat{a}_l} \; \vec{x}_2 \;\right\} \\
& \prod_{l=1}^{Q} \quad _{s_l} \; \vec{x}_2-\vec{x}_1 \;\; ,
\end{aligned}$$

s_l $\qquad$ l

IV. SSF Estimator SNR Numerical Results

4.1 SSF Estimator SNR Numerical Results for the DIMM Geometry

$\vec{\rho}$

$\sqrt{N}$ N

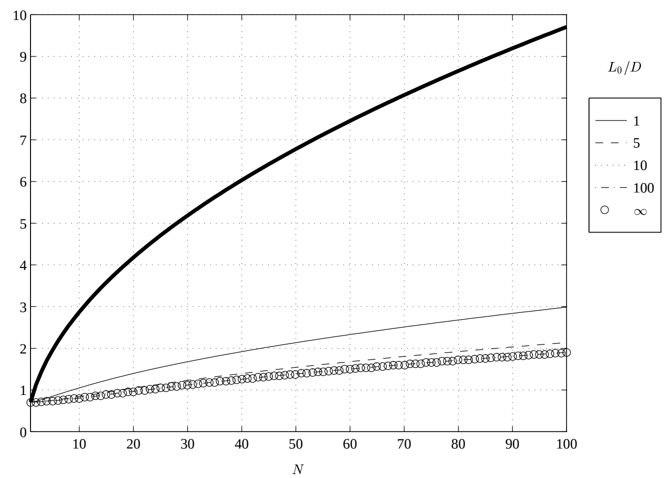
$\sqrt{N}$ $\sqrt{N}$

N

L_0/D

$|\vec{v}\tau|$

. D



N

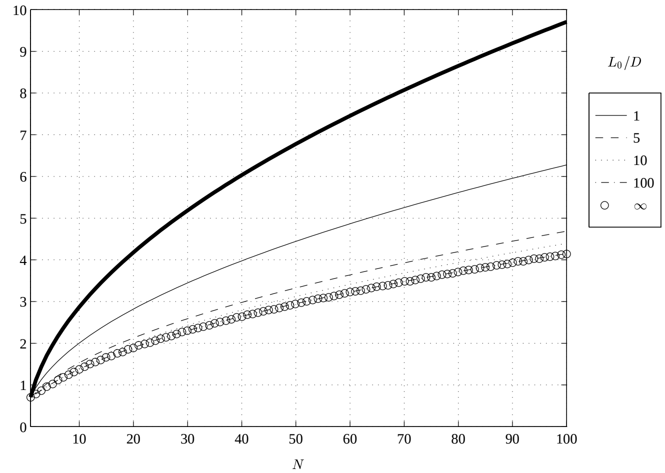
$\vec{\rho}$. D, . $^\circ$

α .

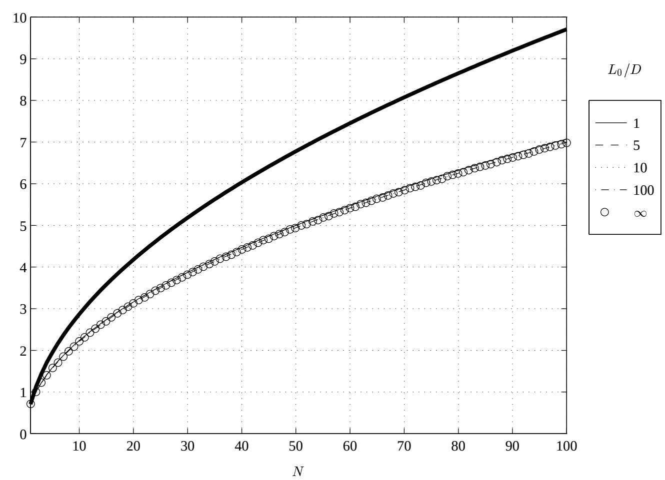
$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/\ _s$ L_0/D

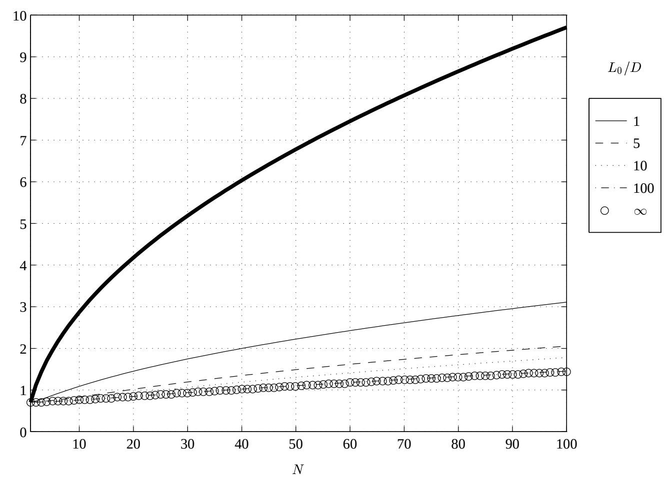
∞



N
$\vec{\rho}$. D, . $^\circ$
α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s L_0/D
∞



N
$\vec{\rho}$. D, . $^\circ$
α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s L_0/D
∞



N

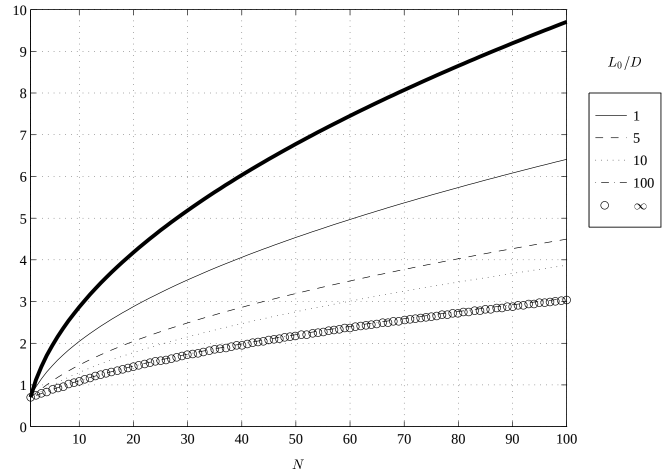
$\vec{\rho}$. D, . $^\circ$

α .

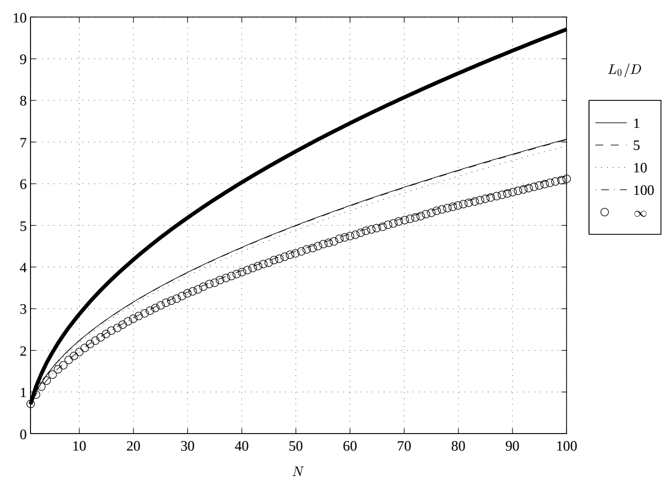
$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s L_0/D

∞

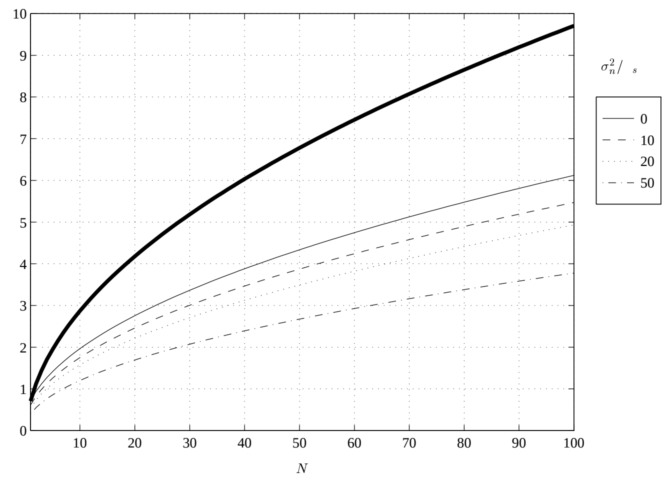


N
$\vec{\rho}$. D, . $^\circ$
α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s L_0/D
∞



N
$\vec{\rho}$. D, . $^\circ$
α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s L_0/D
∞

$\sigma_n^2/\quad s$



N

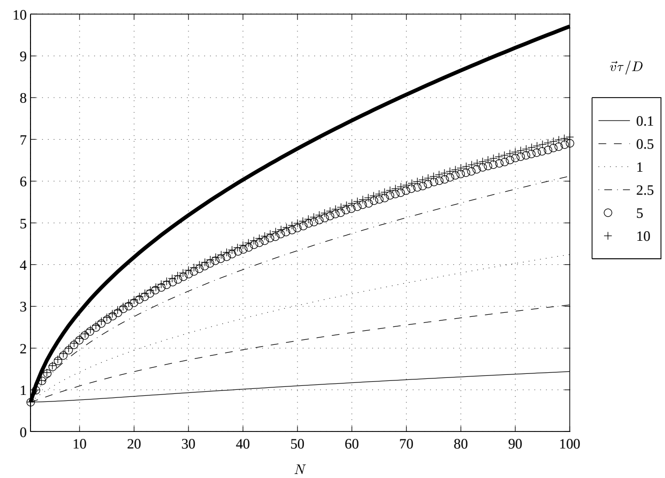
$\vec{\rho}$. D, . $\circ$

α .

$\vec{v}\tau$. D, . $\circ$ L_0/D ∞

$\sigma_n^2/\quad s$

$\sqrt{N}$



N

$\vec{\rho}$. D, . $^\circ$

α .

$\vec{v}$. $^\circ$ L_0/D ∞

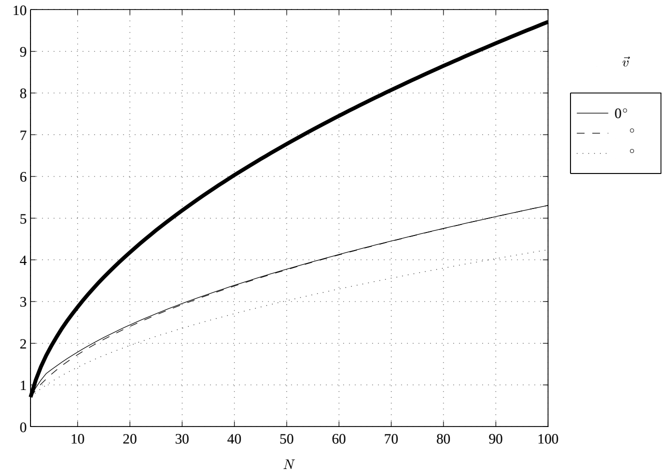
$\sigma_n^2/$ $_s$

. D D

$\sqrt{N}$

$^\circ$

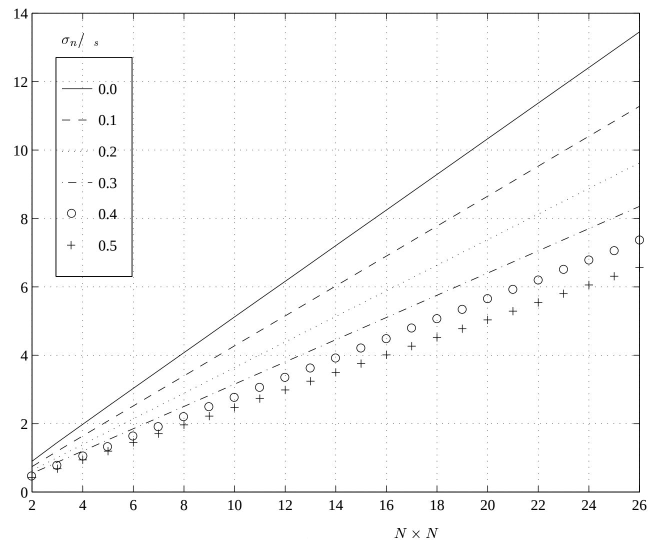
$^\circ$



N
$\vec{\rho}$. D, . °
α .
$|\vec{v}|$. D L_0/D ∞
$\sigma_n^2/$ s
$\vec{v}$
$\sqrt{N}$

4.2 SSF Estimator SNR Numerical Results for non-DIMM Geometry

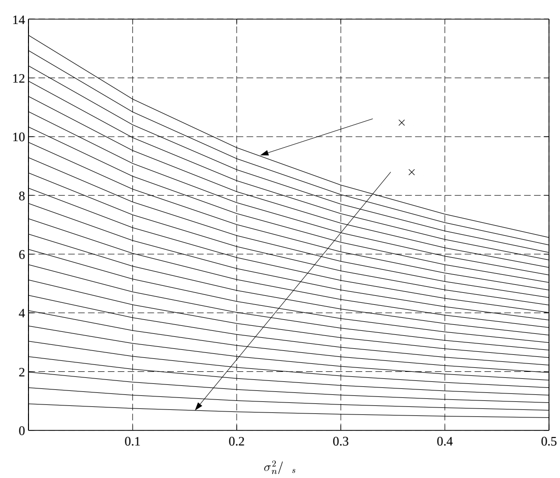
$\vec{\rho}$



$\sigma_n^2/\ \ s$

$\vec{\rho}$ $D,$ α $/$
L_0/D ∞

$\times$



$\sigma_n^2/\ \ s$

$\vec{\rho}$ $D,$
α $/$ L_0/D ∞
$\times$ $\times$

V. Results, Conclusions and Recommendations

5.1 Summary of Theoretical Development

s

s

s

D_s

σ_n^2

ρ_0

5.2 Summary of Numerical Results

5.3 Recommendation for Future Research

- [1]

-

[1] This simulation should include a phase screen generator capable of including outer scale, power law, and temporal effects. This author has developed and tested a phase screen generator, with all these capabilities, based on Eqn. (2.18).

s

$$\mathcal{E} \quad a_j a_{j'}^* \ \vec{u} \qquad c_1 \quad \frac{D}{\rho_0} \quad ^{(\alpha-2)} \quad \frac{\frac{\alpha}{2}}{- \quad \frac{2-\alpha}{2}} \quad n \qquad n' \qquad ^{1/2} f_{jj'} \ u, \theta_0, k_0 \ ,$$

$$f_{jj'} \ u, \theta_0, k_0$$
$$- \quad ^{(n+n')/2} \qquad \theta_0 \ I_{2,n+1,n'+1} \quad u, k_0 \qquad j, j'$$
$$- \quad ^{(n+n'+2)/2} I_{0,n+1,n'+1} \quad u, k_0$$
$$- \ - \quad ^{(n+n')/2} \qquad \theta_0 \ I_{2,n+1,n'+1} \quad u, k_0 \qquad j, j' \qquad ,$$
$$- \quad ^{(n+n'+2)/2} I_{0,n+1,n'+1} \quad u, k_0$$

$$k_0 \quad \pi \frac{D}{L_0},$$

$$I_{\kappa,\mu,\nu} \ a, x_0 \qquad _0^{\infty} \frac{x^{-1} J_\kappa \ ax \ J_\mu \ x \ J_\nu \ x}{x^2 \quad {x_0}^2 \ ^{\alpha/2}} \ x,$$

c_1 D L_0 u

θ_0 $J_\kappa \ x$ $J_\mu \ x$ $J_\nu \ x$

κ μ ν

α LoD

		θ_0 °	α	L_0/D

		θ_0 °	α	L_0/D

Z_2 Z_{16}

s

Z_2 Z_{16} s

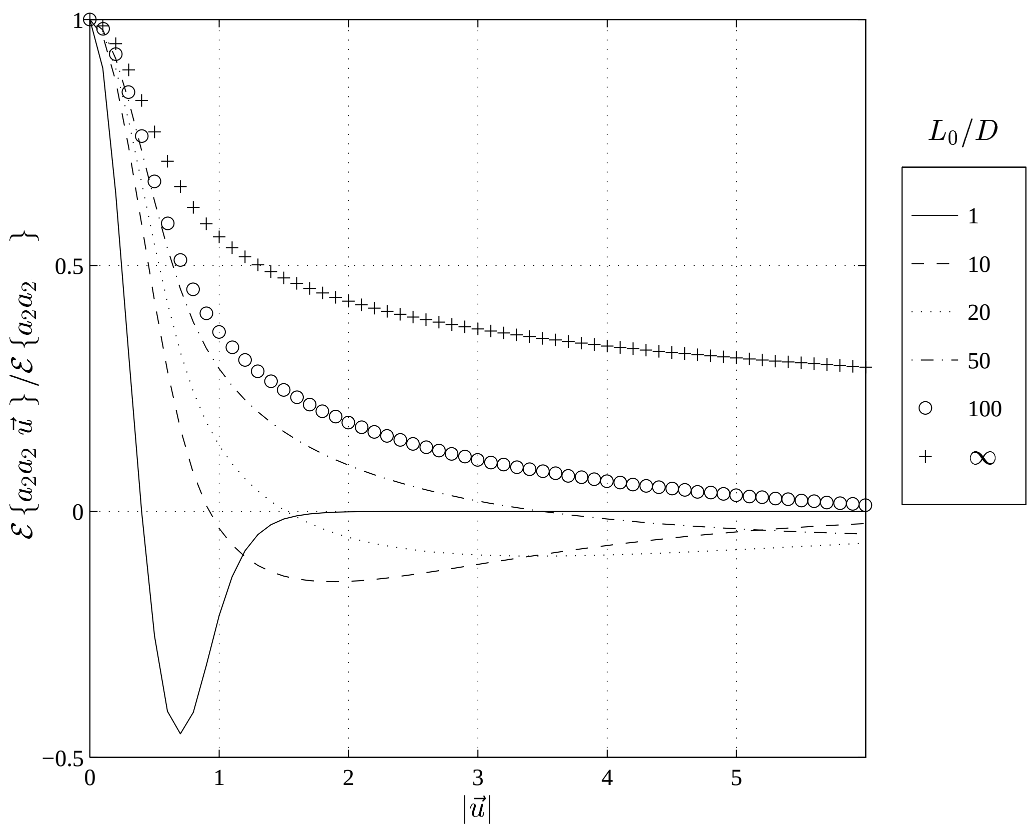
θ_0

j θ_0 $\pi/$

j

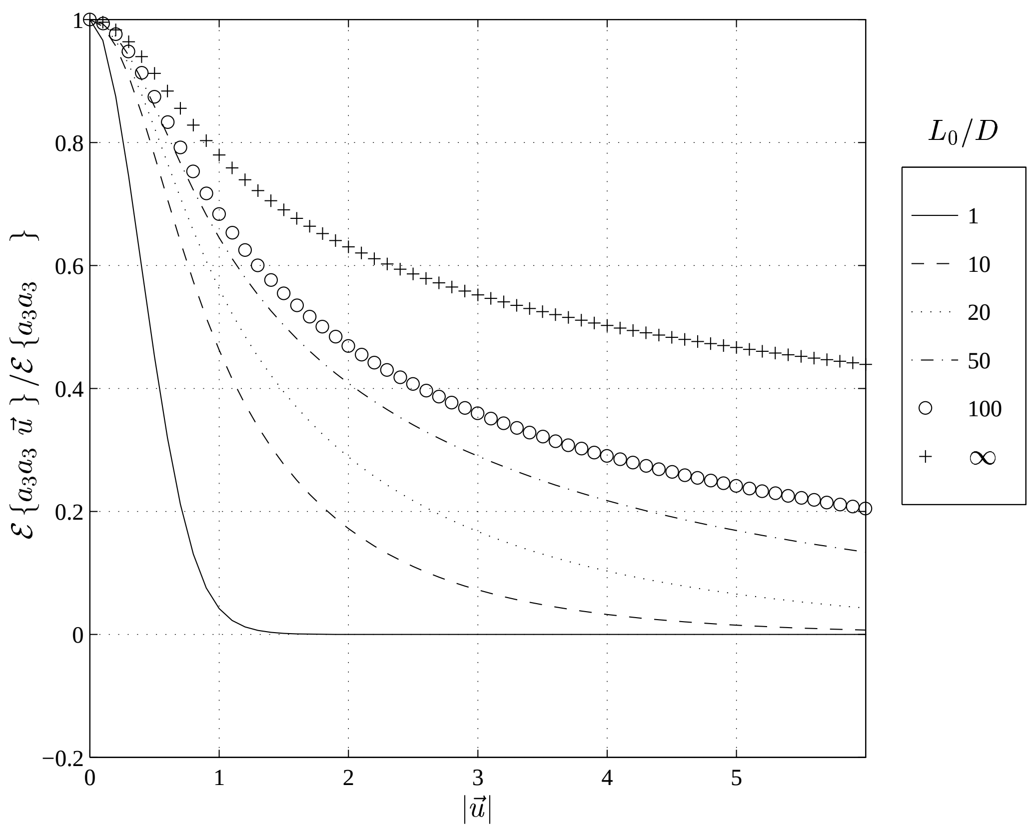
α

L_0/D



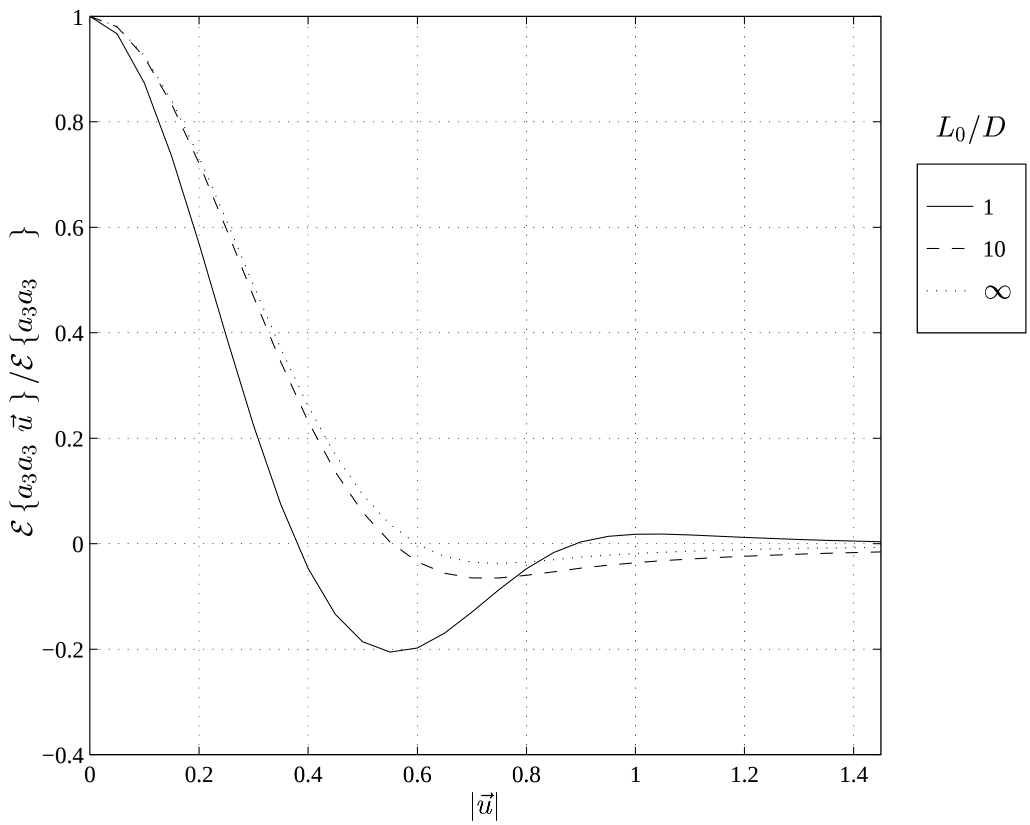
$\mathcal{E}\left\{a_2 a_2\ \vec{u}\ \right\}/\mathcal{E}\left\{a_2 a_2\quad\right\}$

θ_0



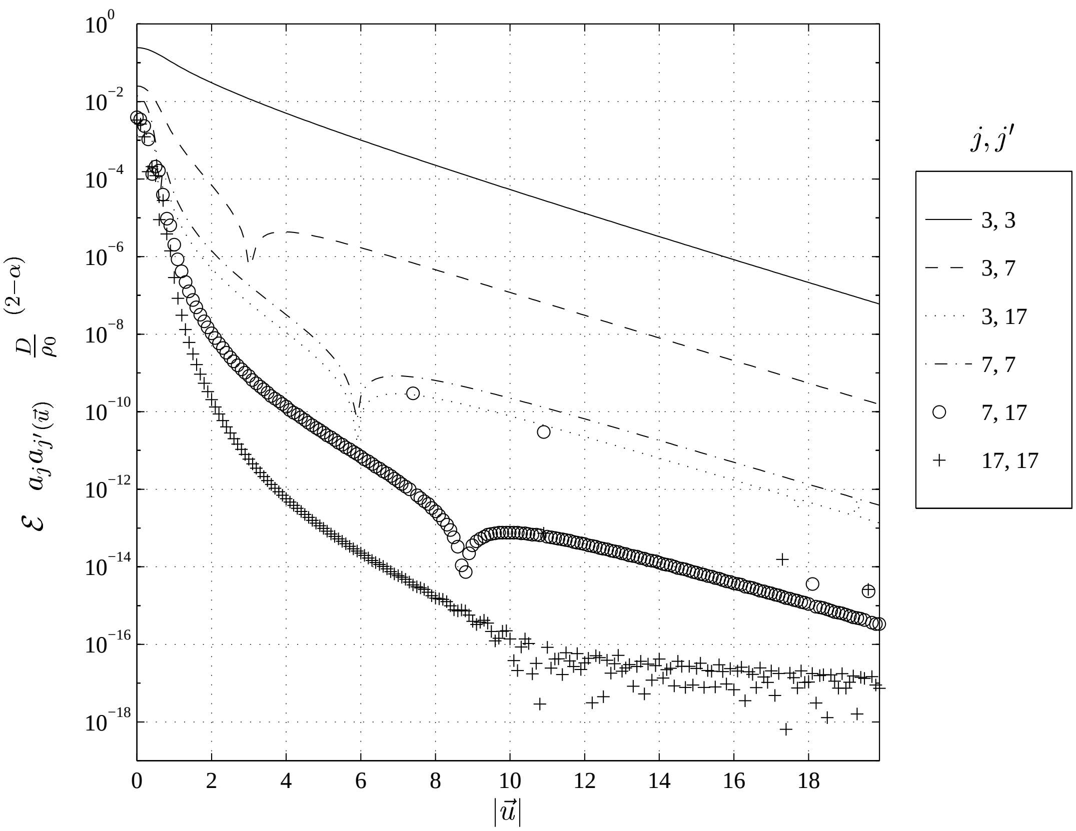
$\mathcal{E}\{a_3 a_3\ \vec{u}\ \}/\mathcal{E}\{a_3 a_3\ \ \}$

θ_0

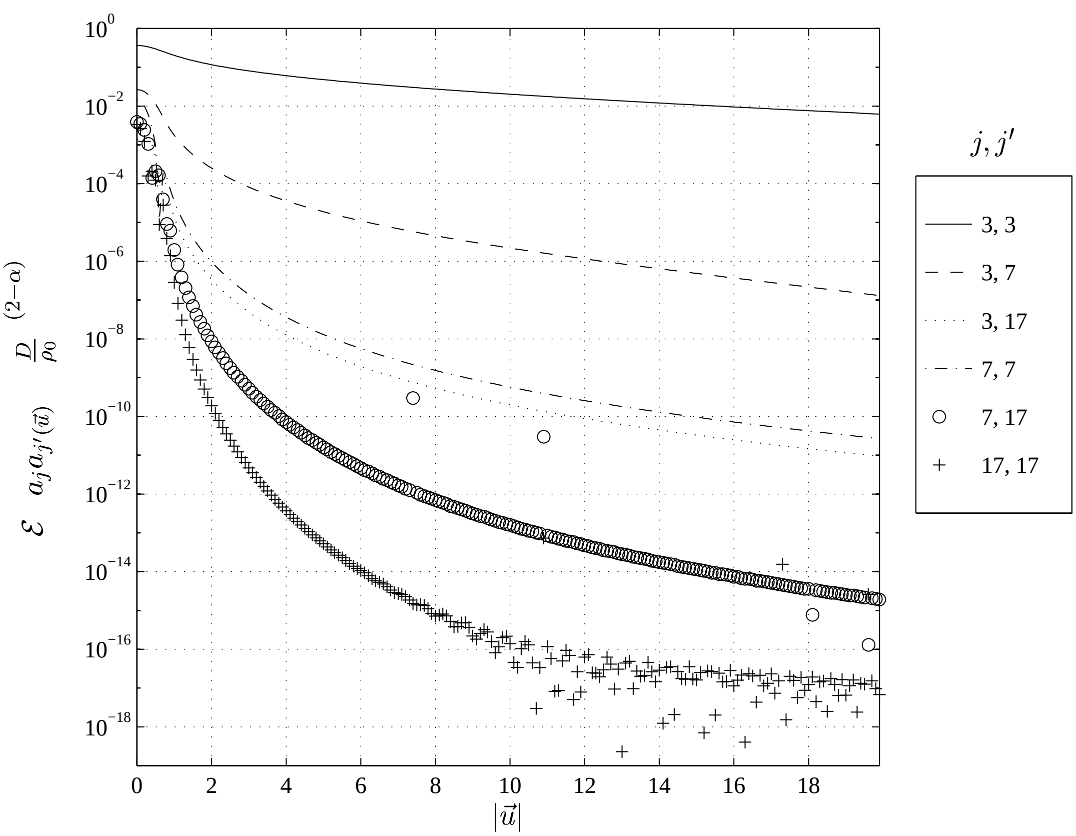


$\mathcal{E}\{a_3a_3\ \vec{u}\ \}/\mathcal{E}\{a_3a_3\ \ \ \}$

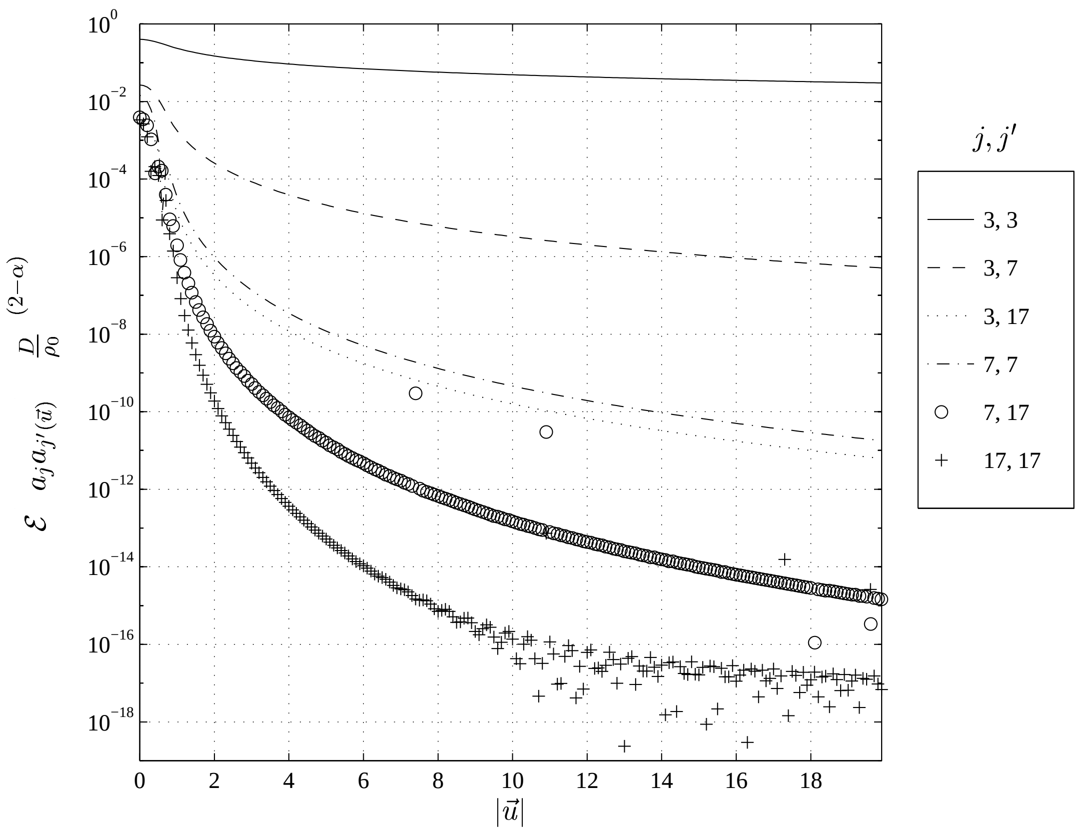
θ_0



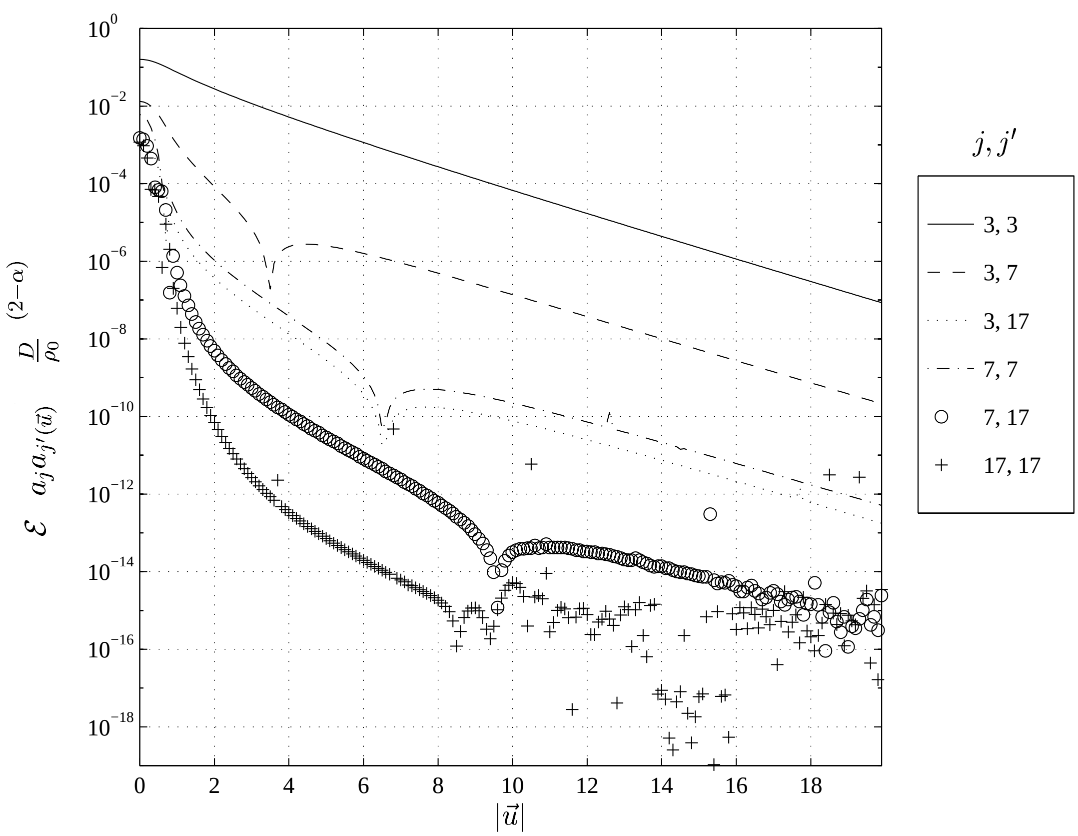
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



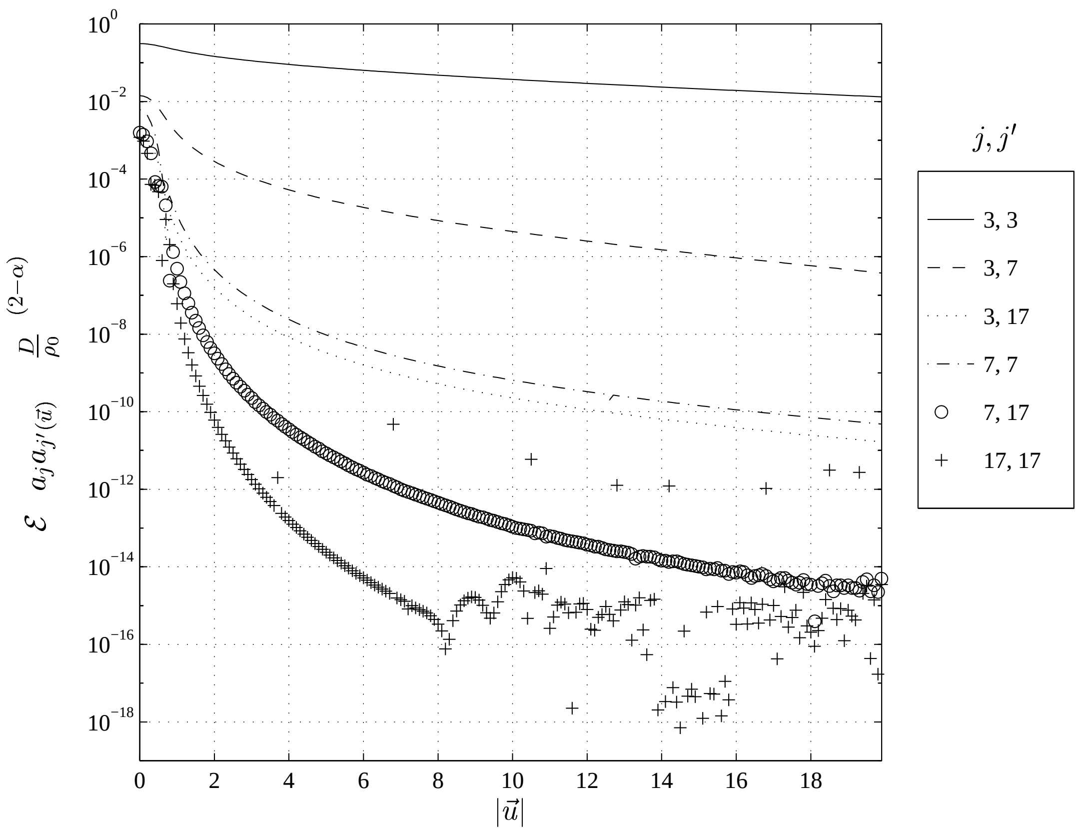
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



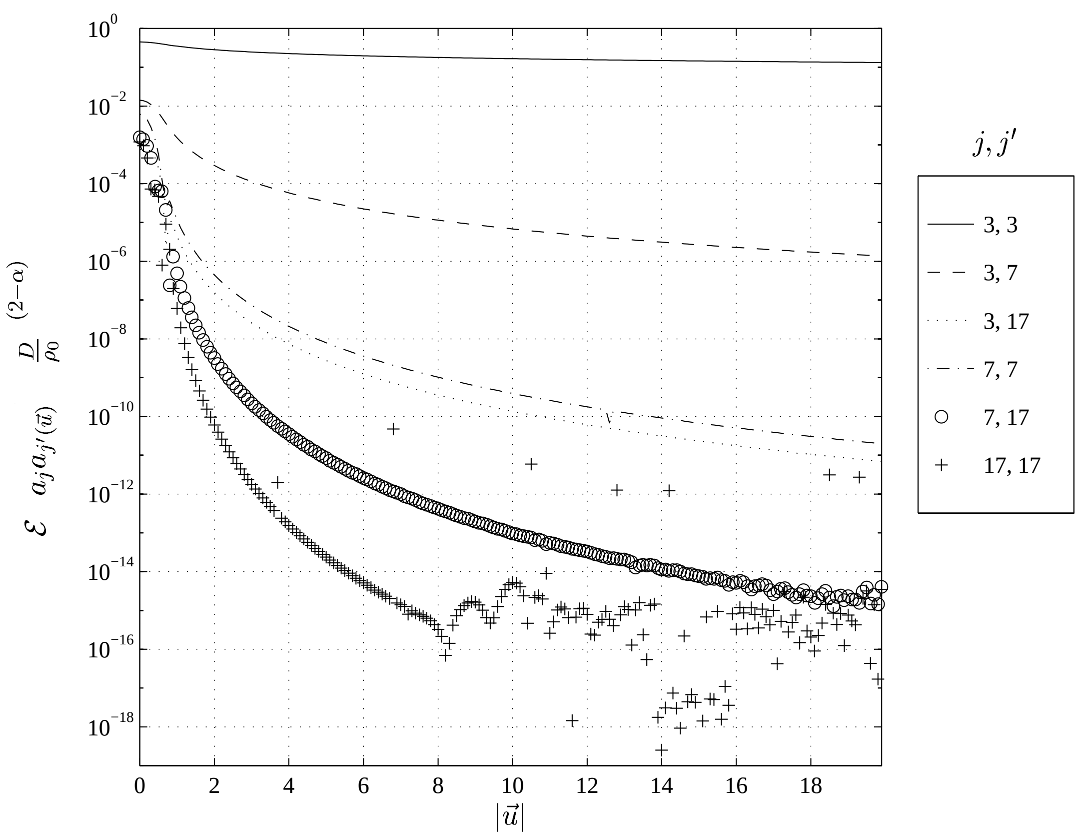
$\mathcal{E}\quad a_j a_{j'(\vec{u})}\quad \frac{D}{\rho_0}\ ^{(2-\alpha)}\qquad \theta_0\quad {}^\circ\ L_0/D\quad \infty\ \ \alpha\quad .$



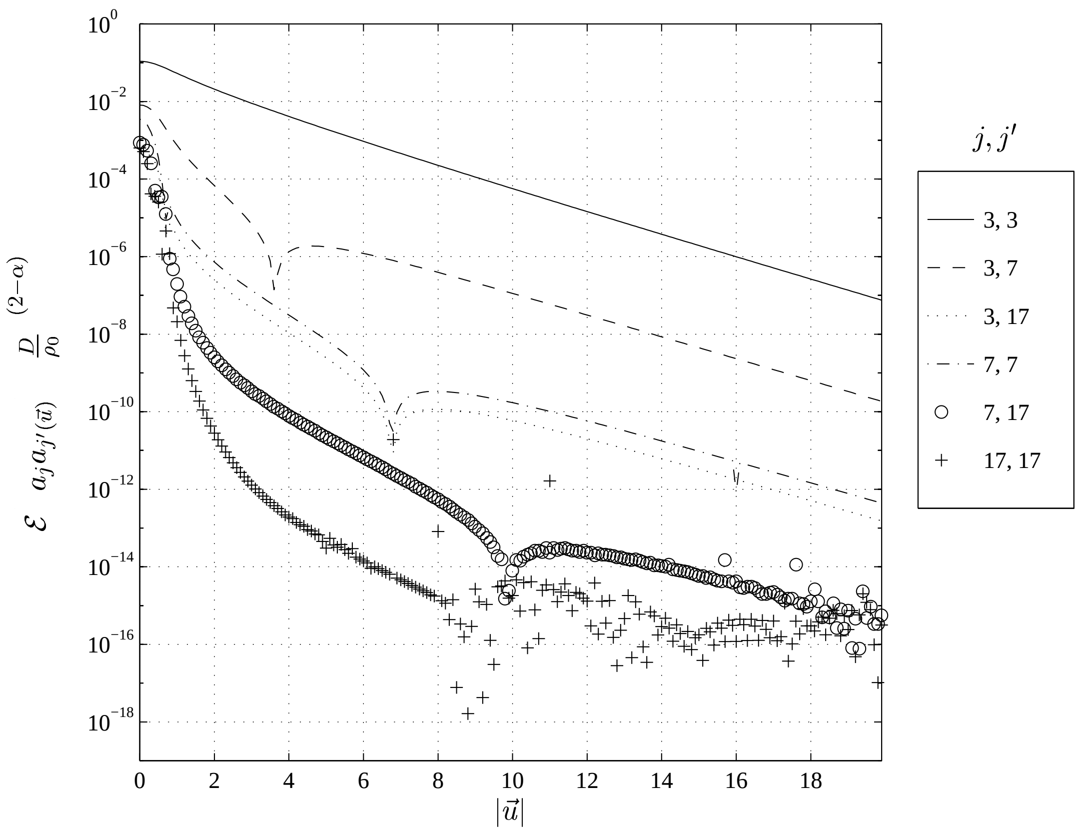
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $\circ$ L_0/D . α .



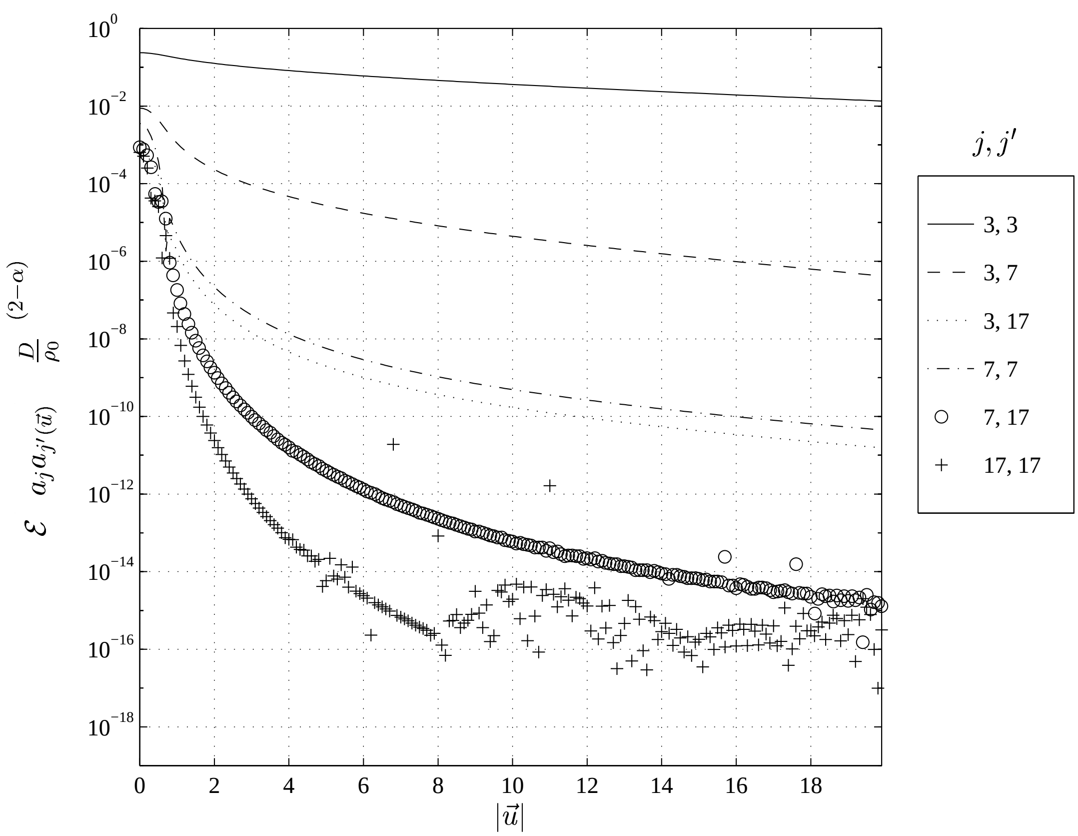
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



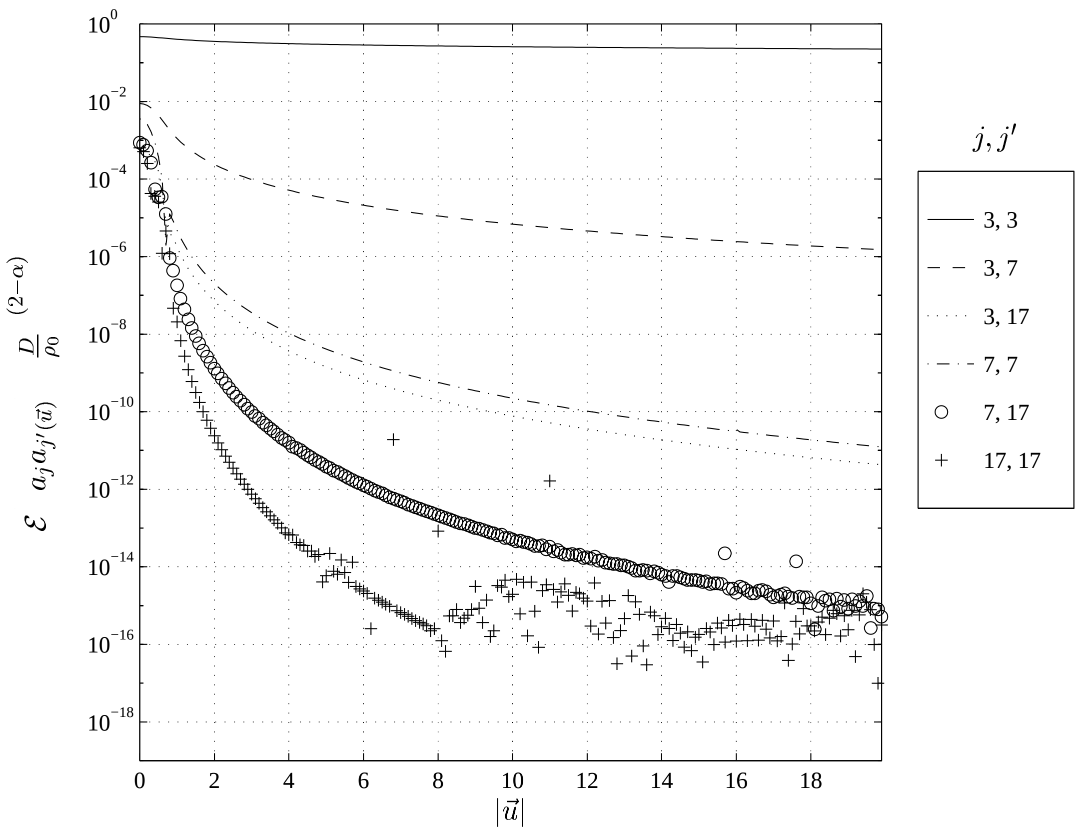
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



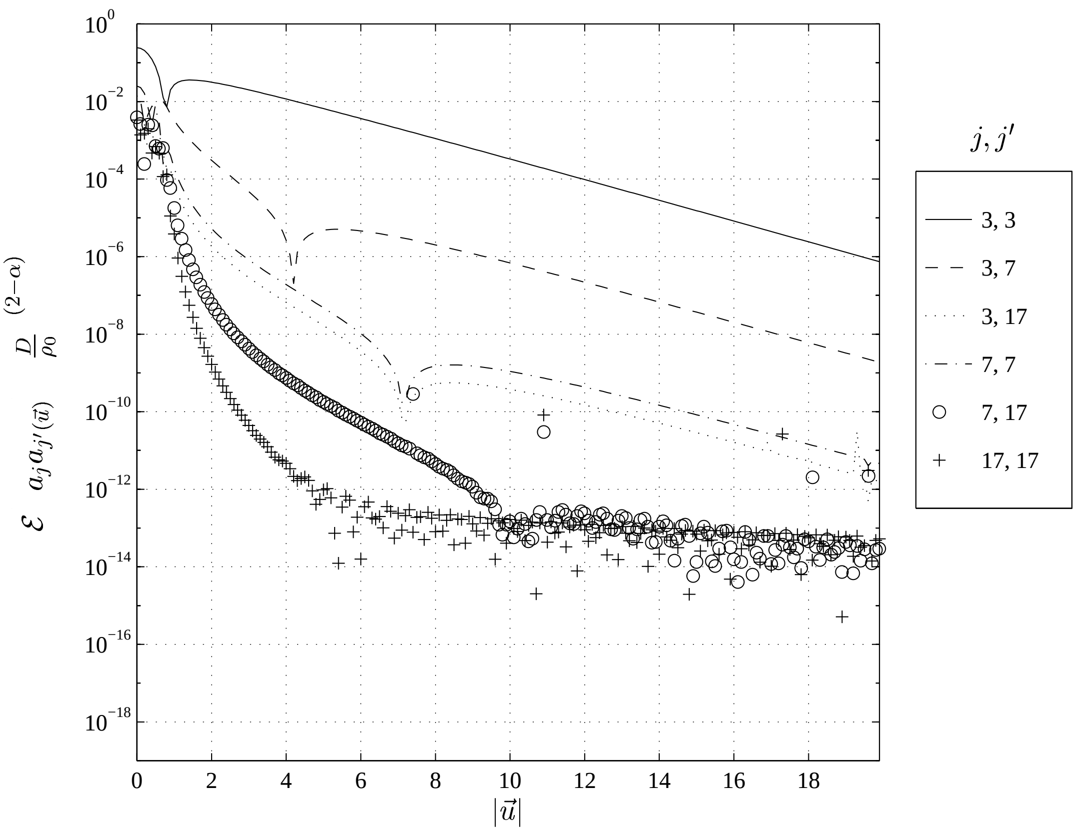
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $\circ$ L_0/D . α .



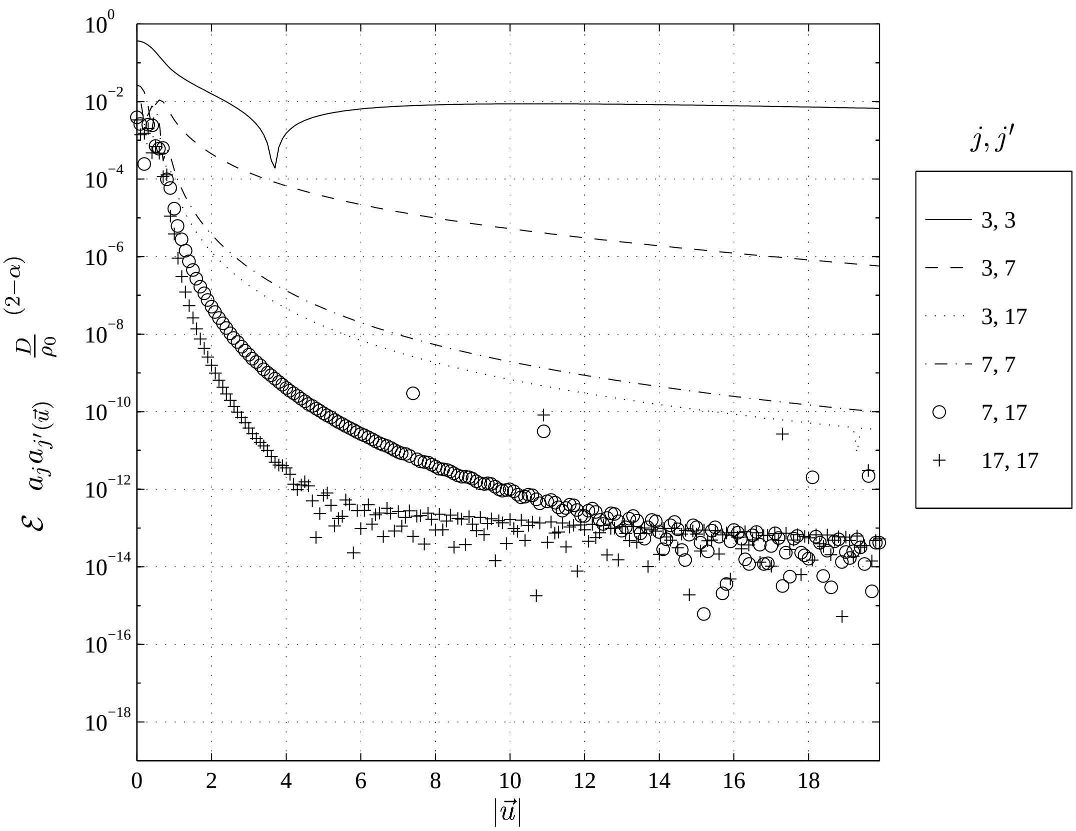
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



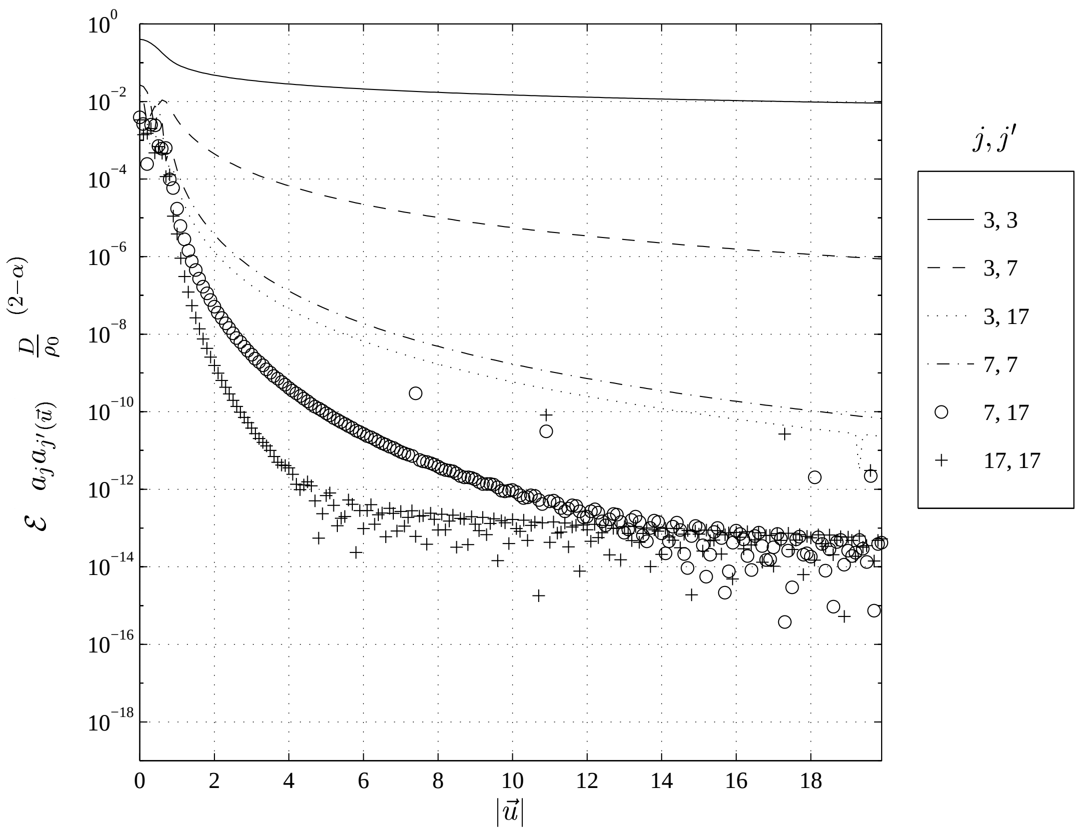
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



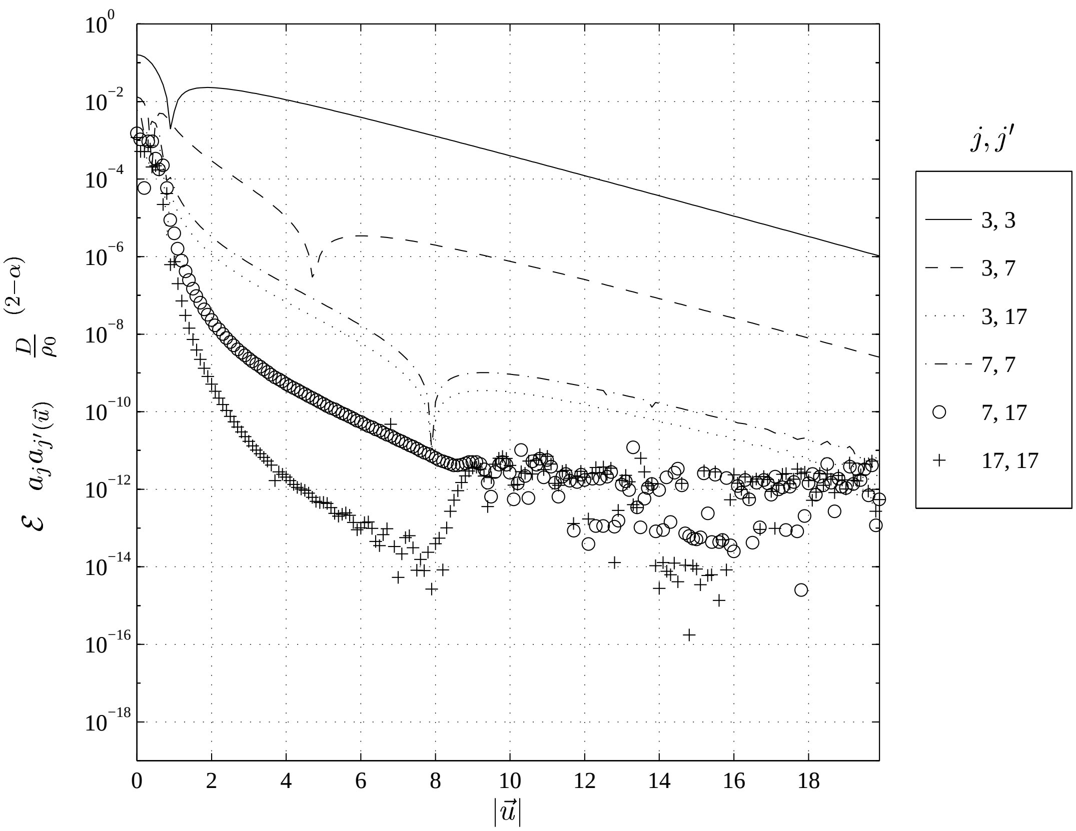
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



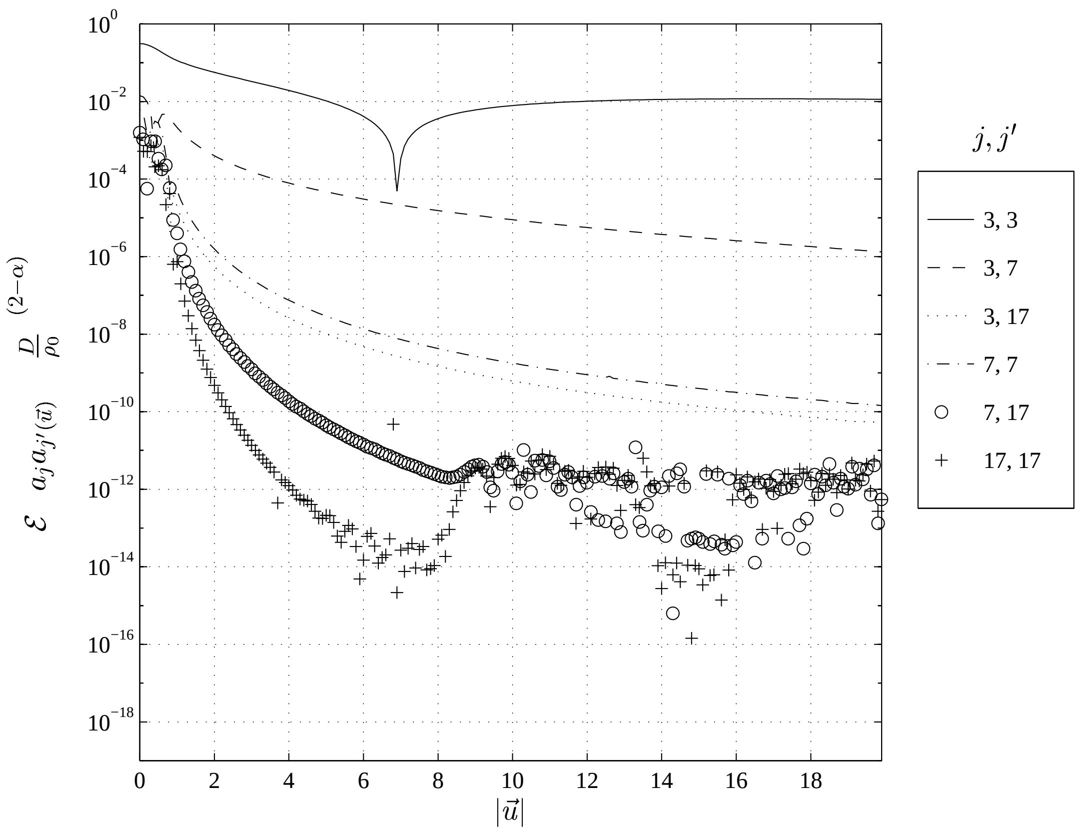
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



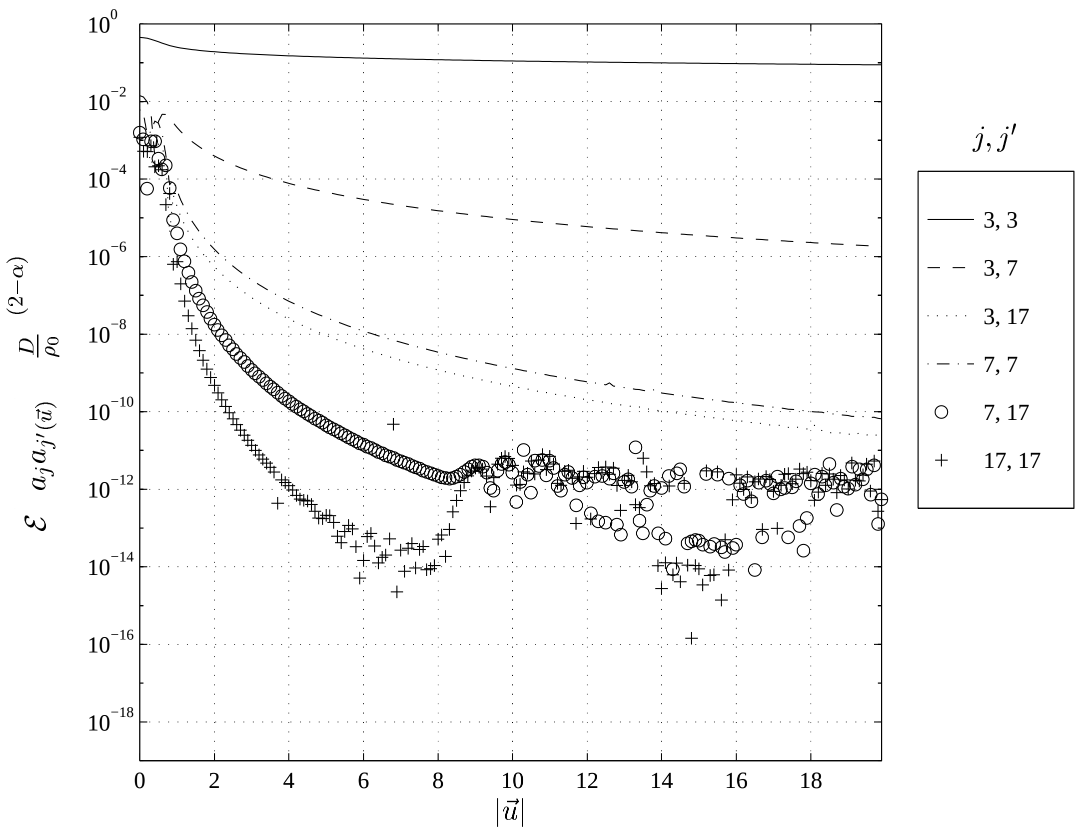
$\mathcal{E}\quad a_j a_{j'(\vec{u})}\quad \frac{D}{\rho_0}^{\,(2-\alpha)}\qquad \theta_0\quad {}^\circ\ L_0/D\quad \infty\ \ \alpha\quad .$



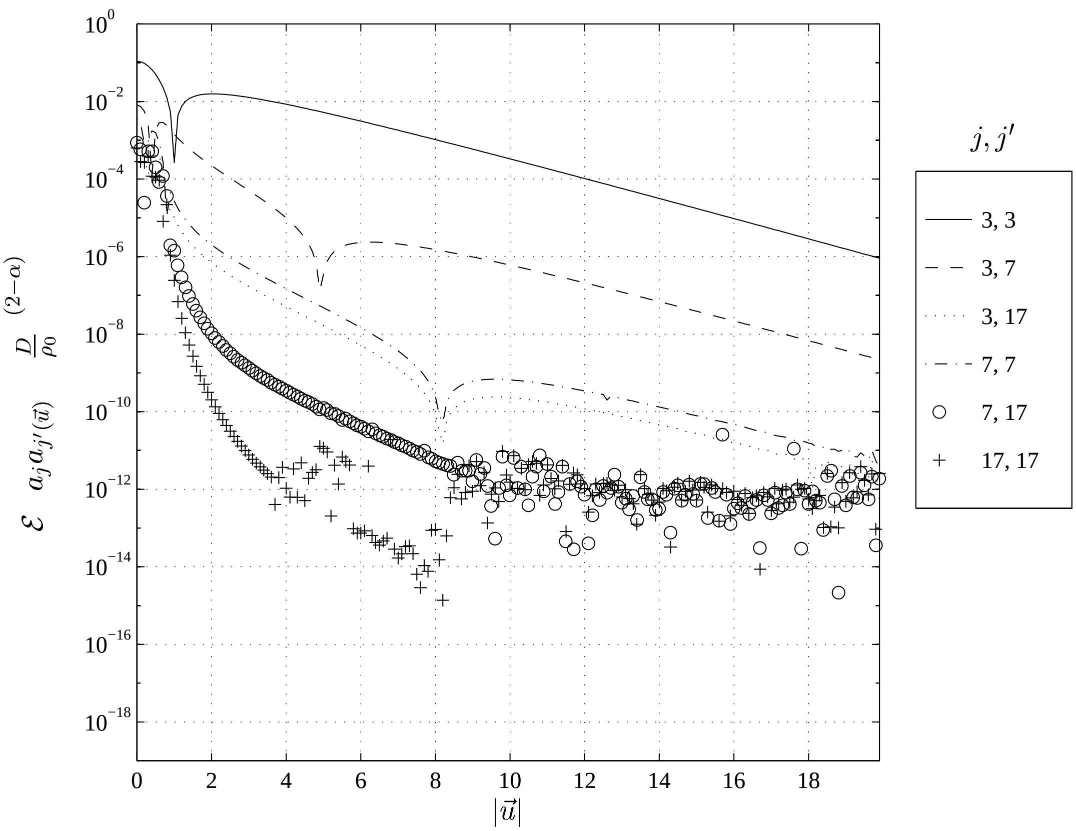
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



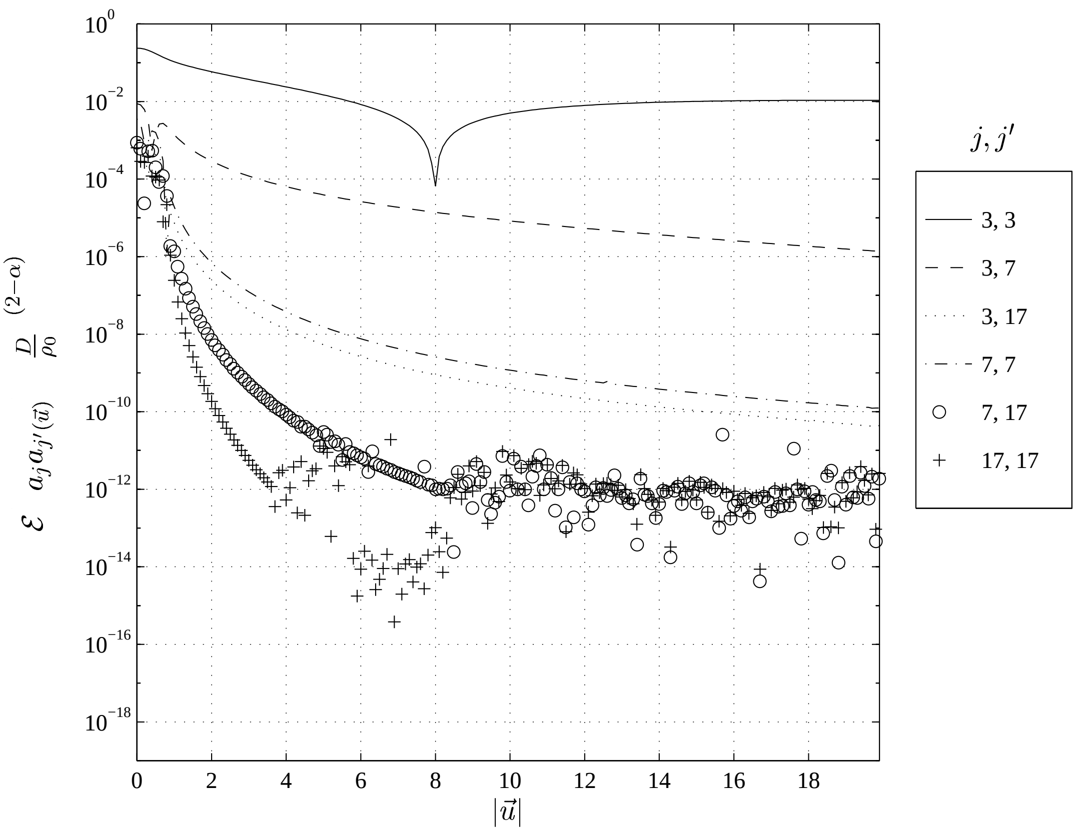
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



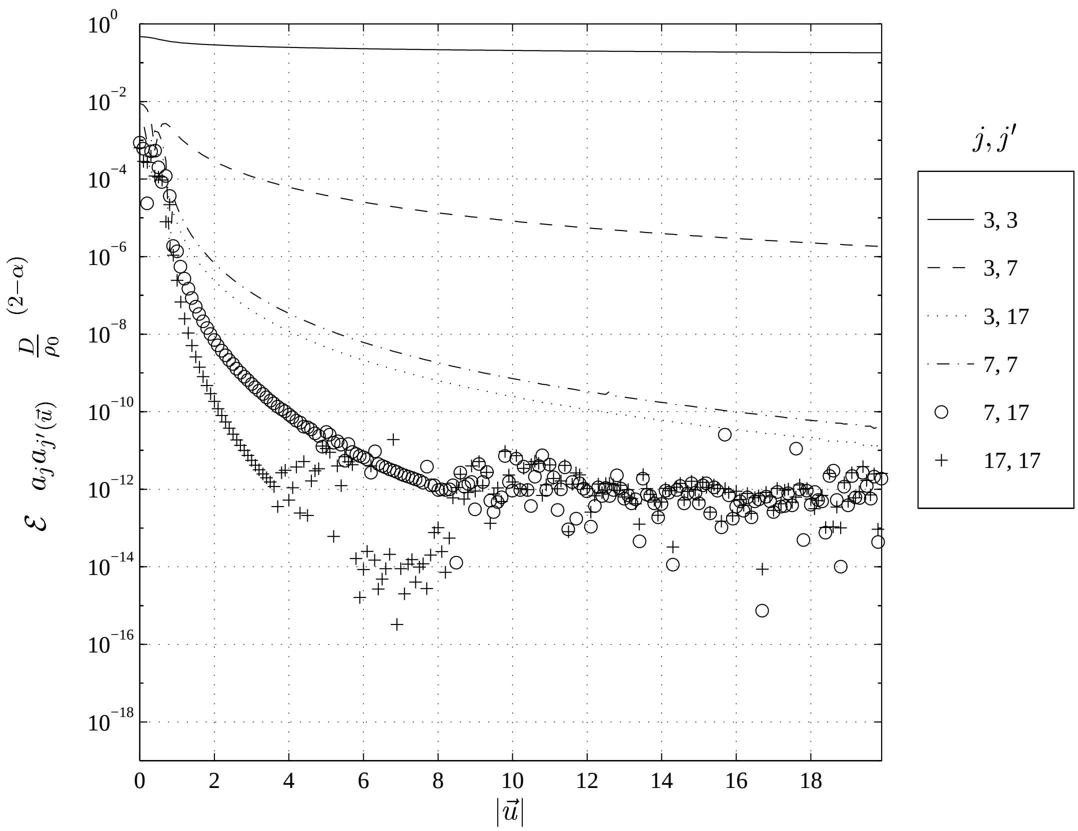
$\mathcal{E} \quad a_j a_{j'(\vec{u})} \quad \frac{D}{\rho_0} \quad ^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



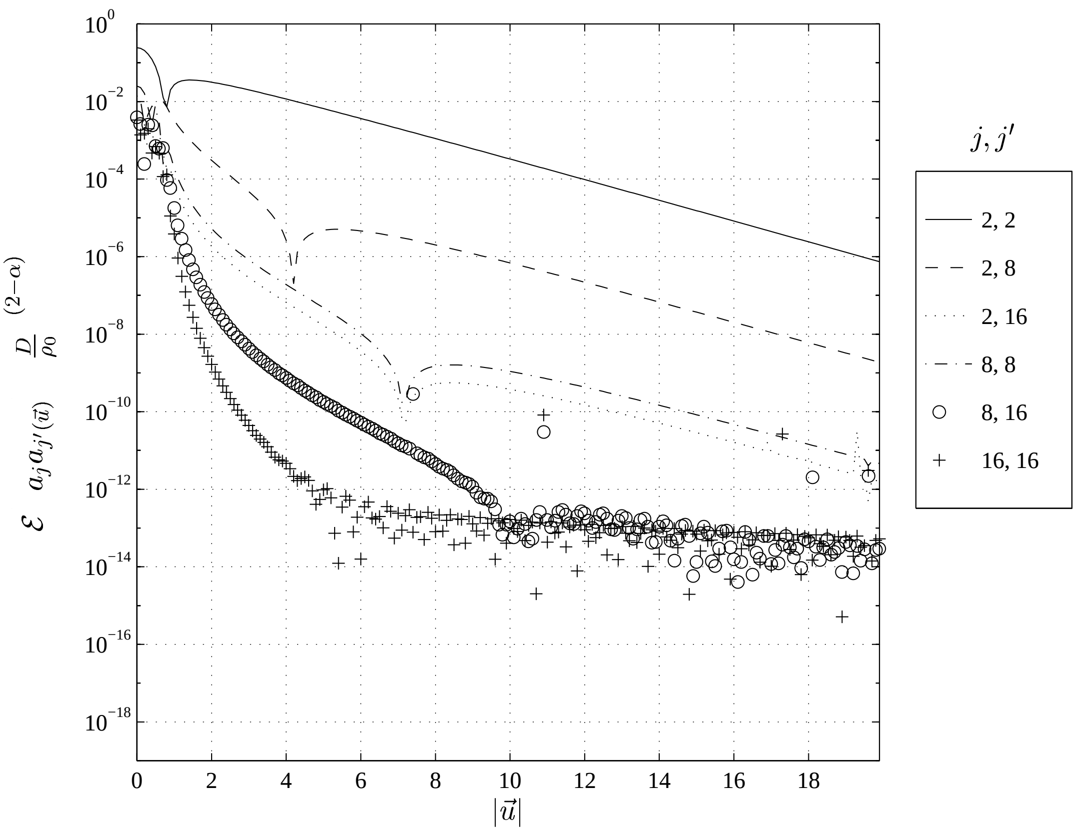
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



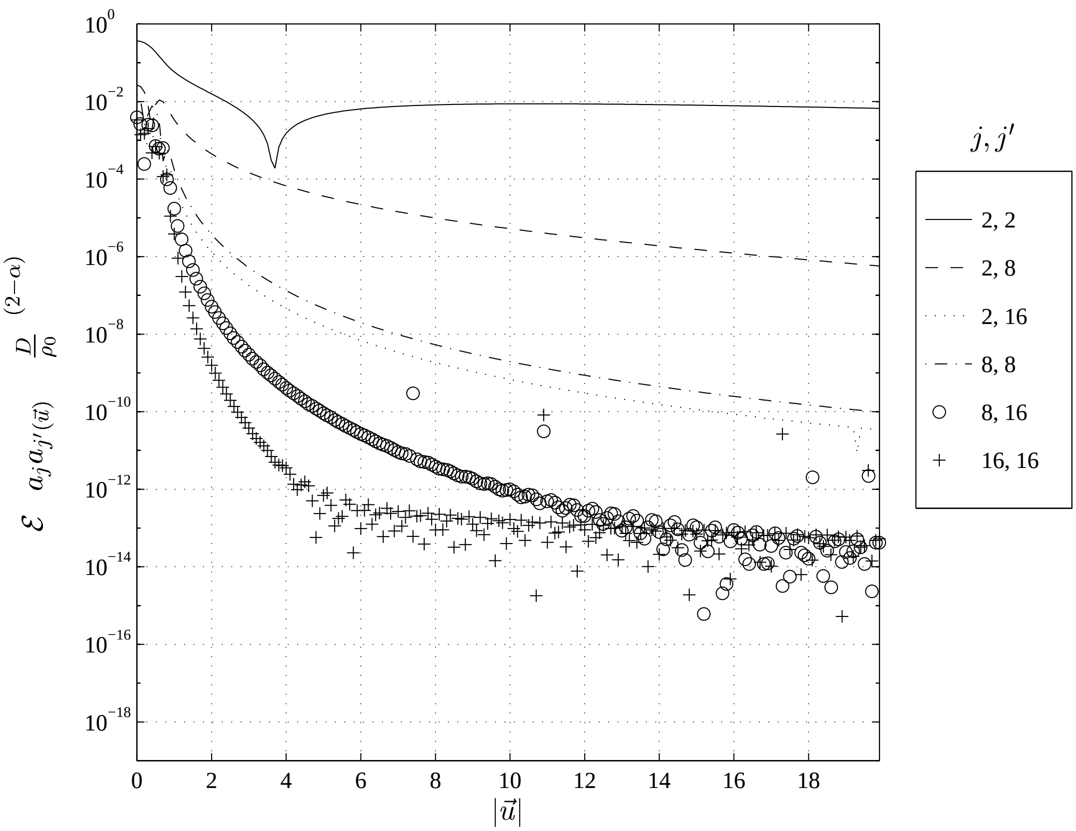
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



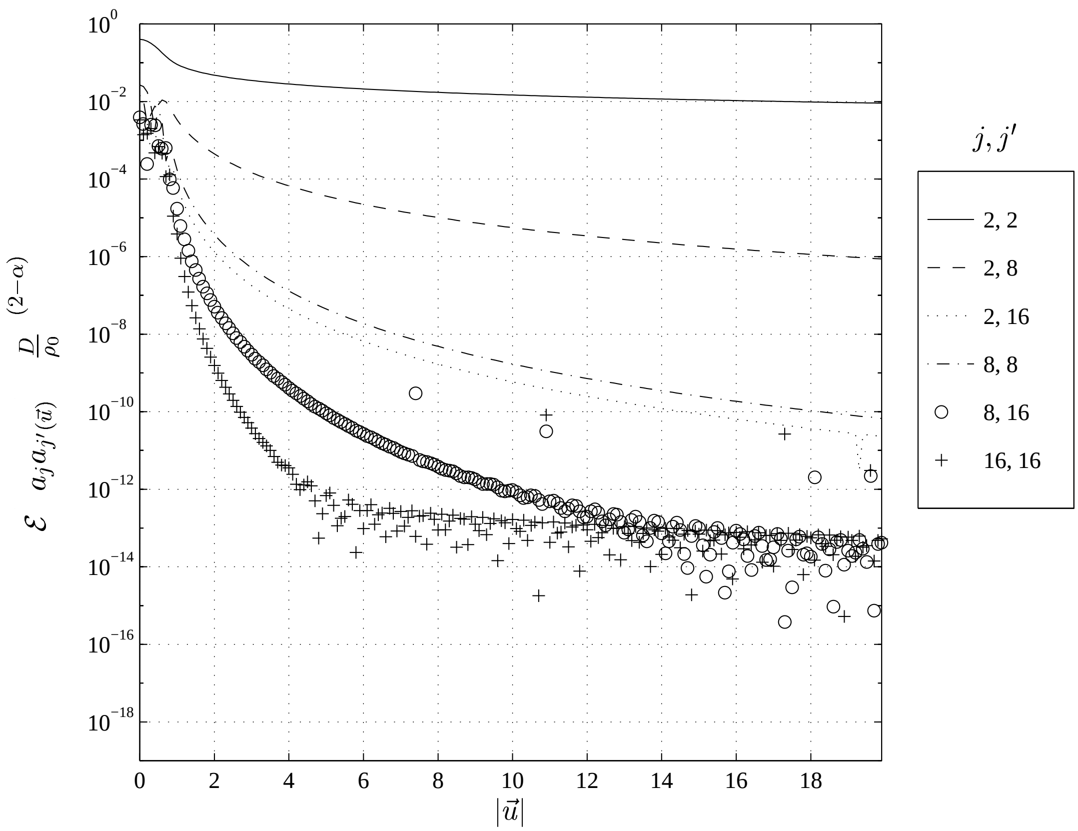
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



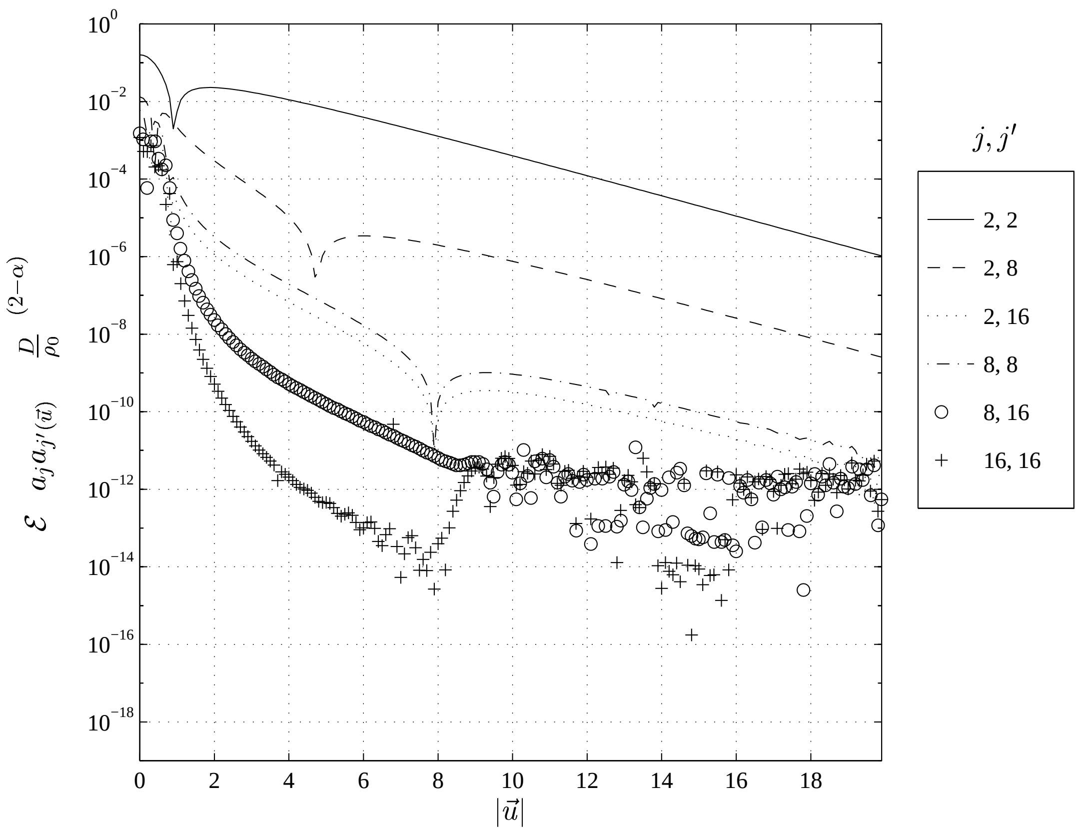
$\mathcal{E} \quad a_j a_{j'(\vec{u})} \quad \frac{D}{\rho_0} \quad ^{(2-\alpha)}$ θ_0 $\circ$ L_0/D . α .



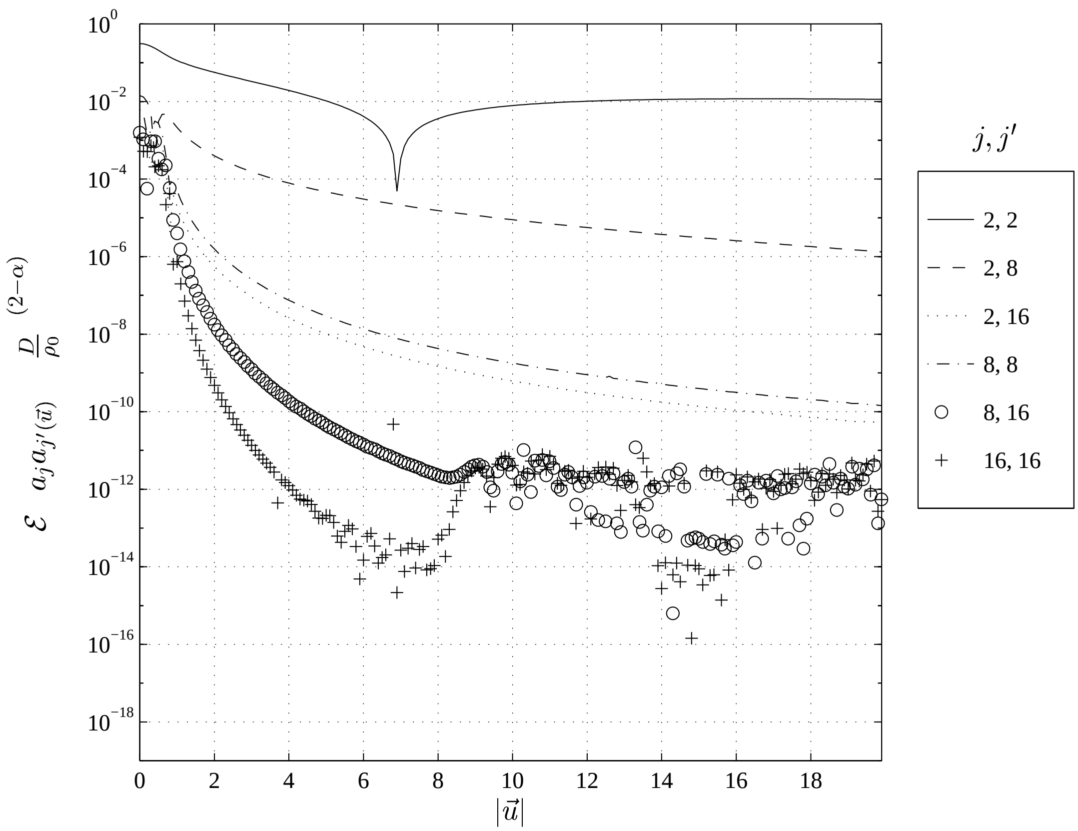
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



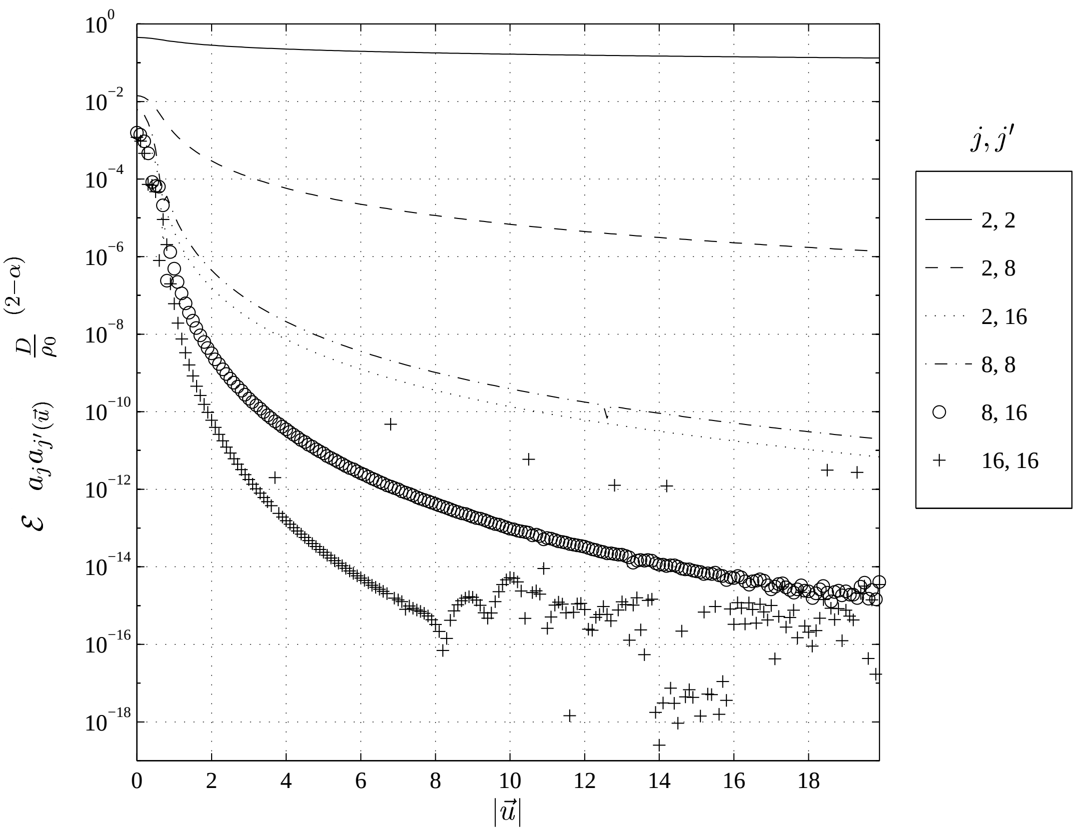
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



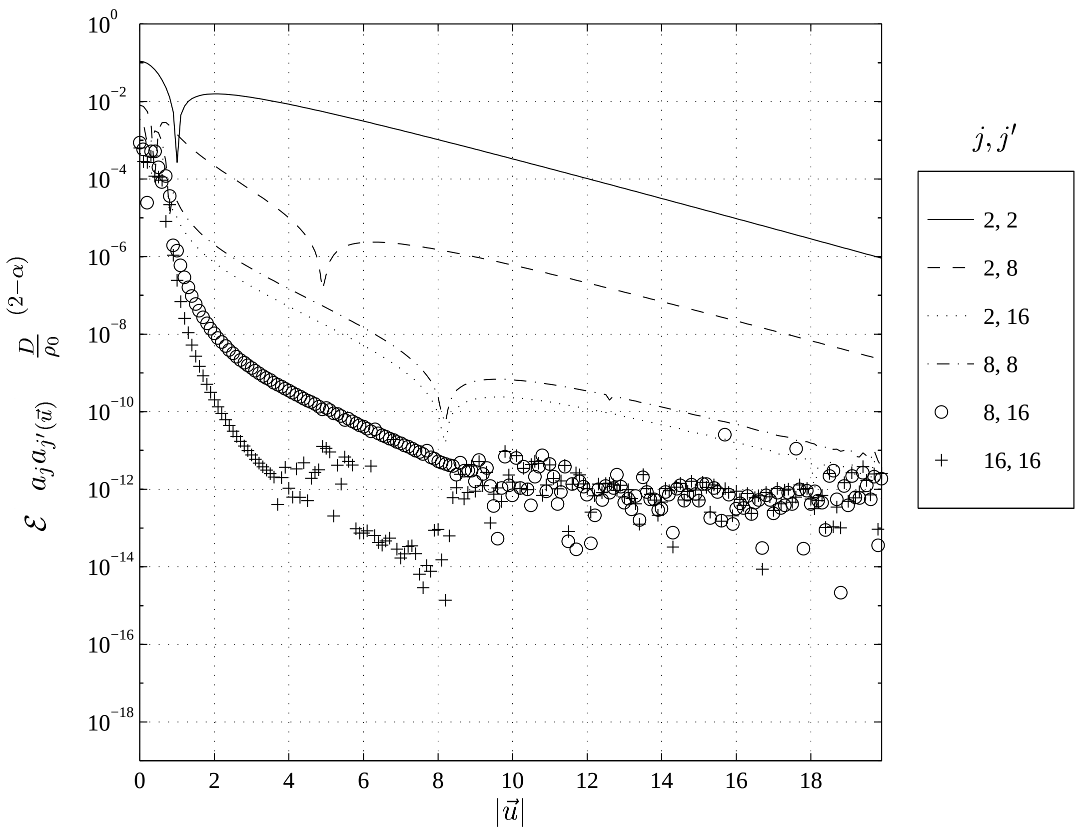
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



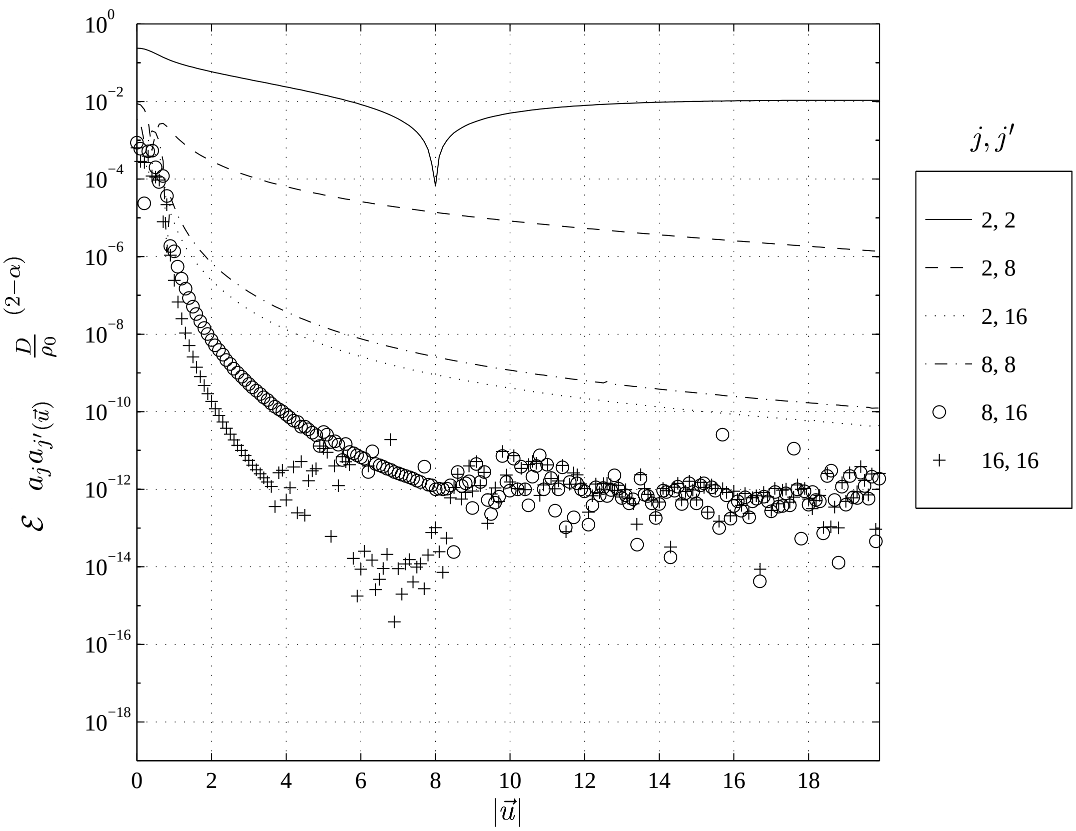
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



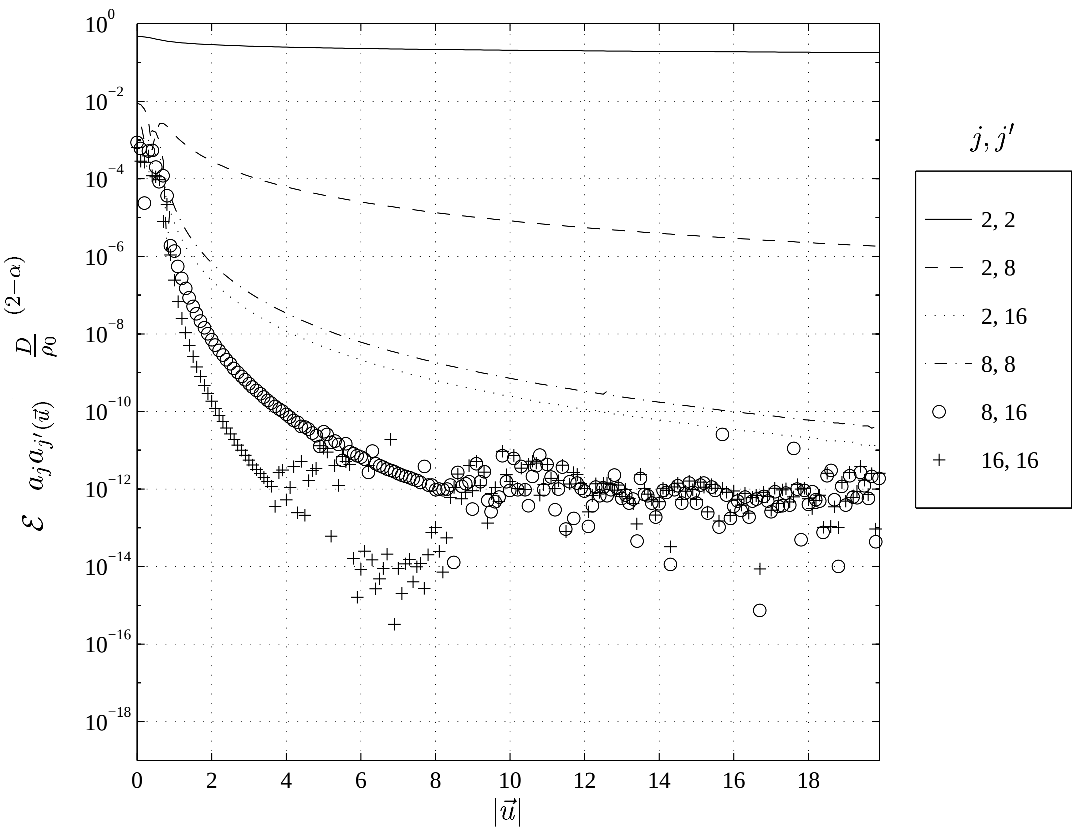
$\mathcal{E}\quad a_j a_{j'(\vec{u})} \quad \frac{D}{\rho_0}^{(2-\alpha)} \qquad \theta_0 \quad {}^\circ \; L_0/D \quad \infty \;\; \alpha \quad .$



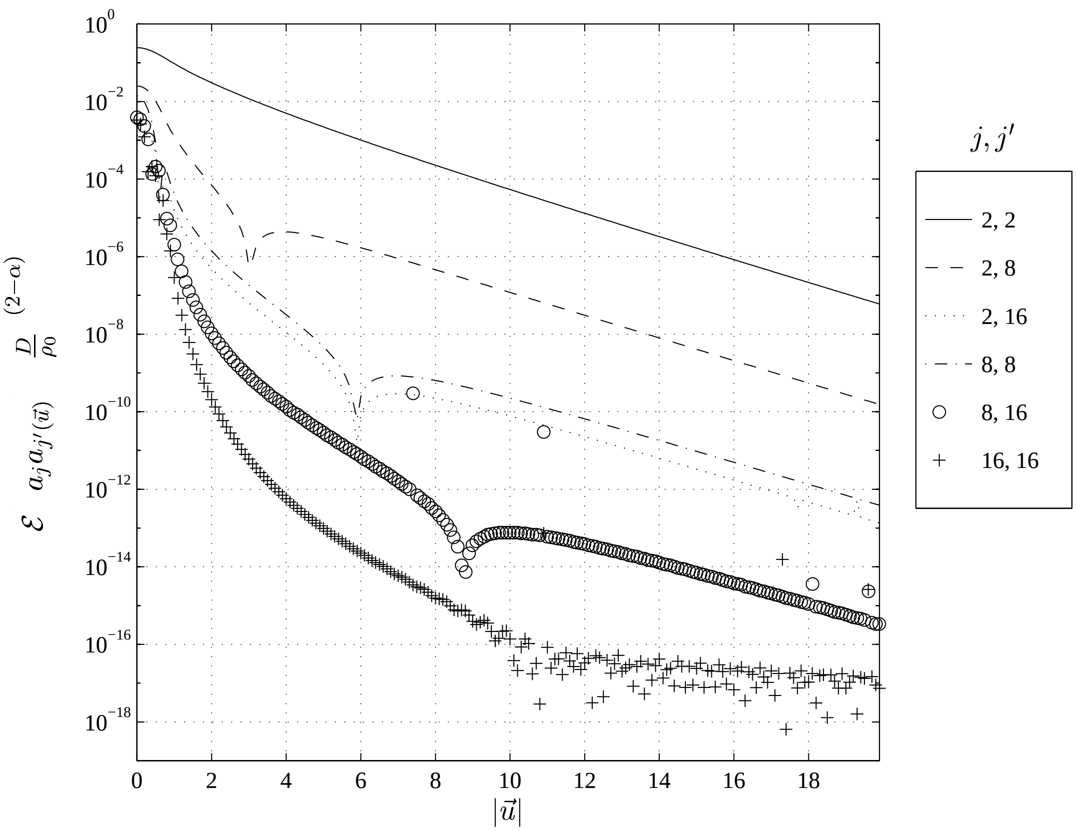
$\mathcal{E} \quad a_j a_{j'(\vec{u})} \quad \frac{D}{\rho_0} \ ^{(2-\alpha)}$ θ_0 $\circ$ L_0/D . α .



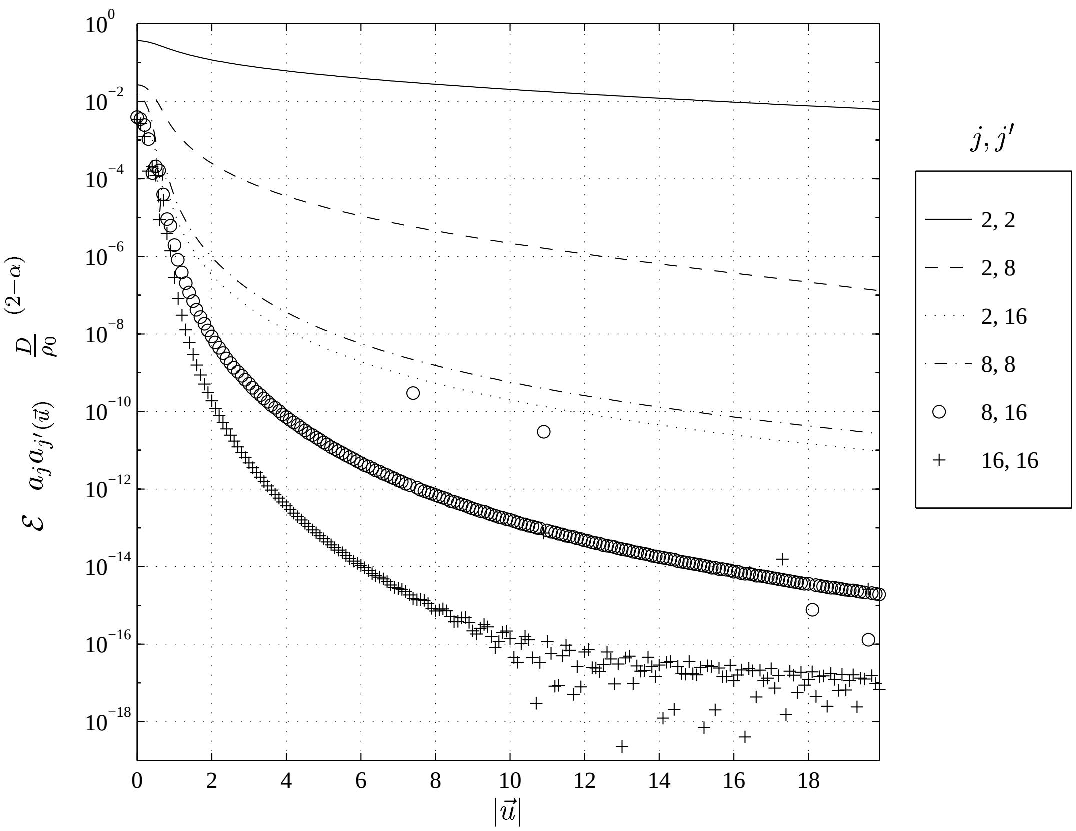
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



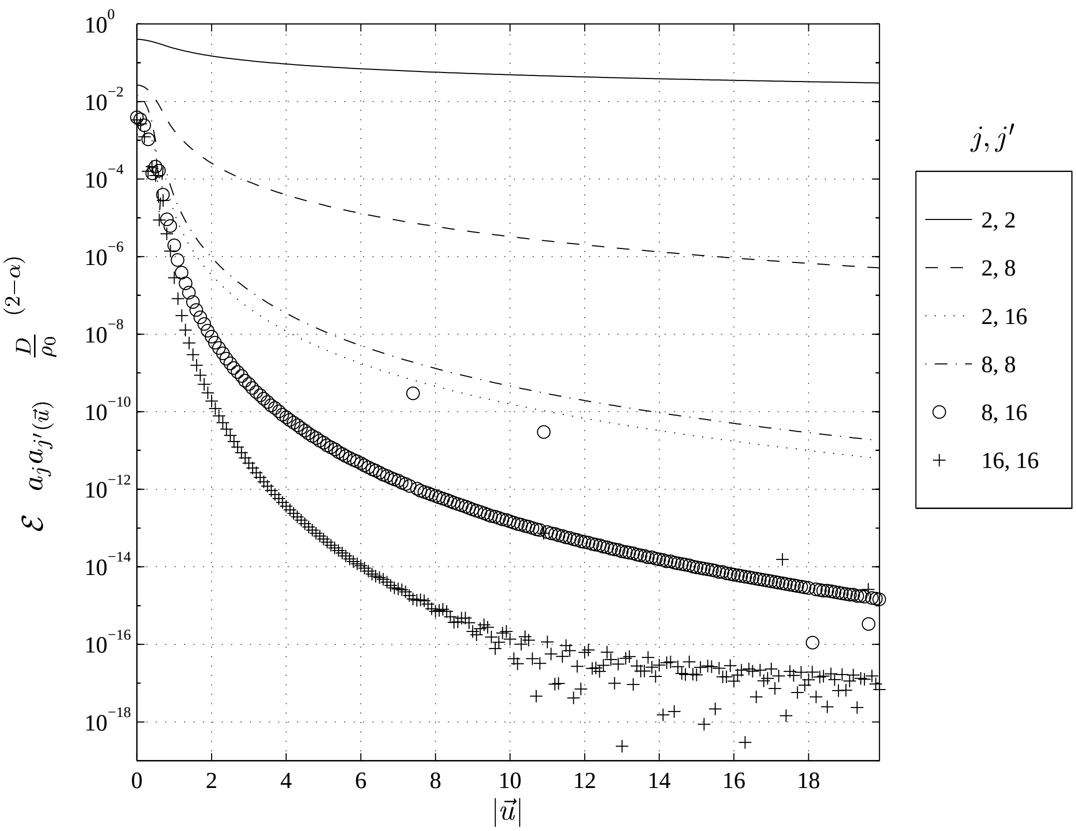
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



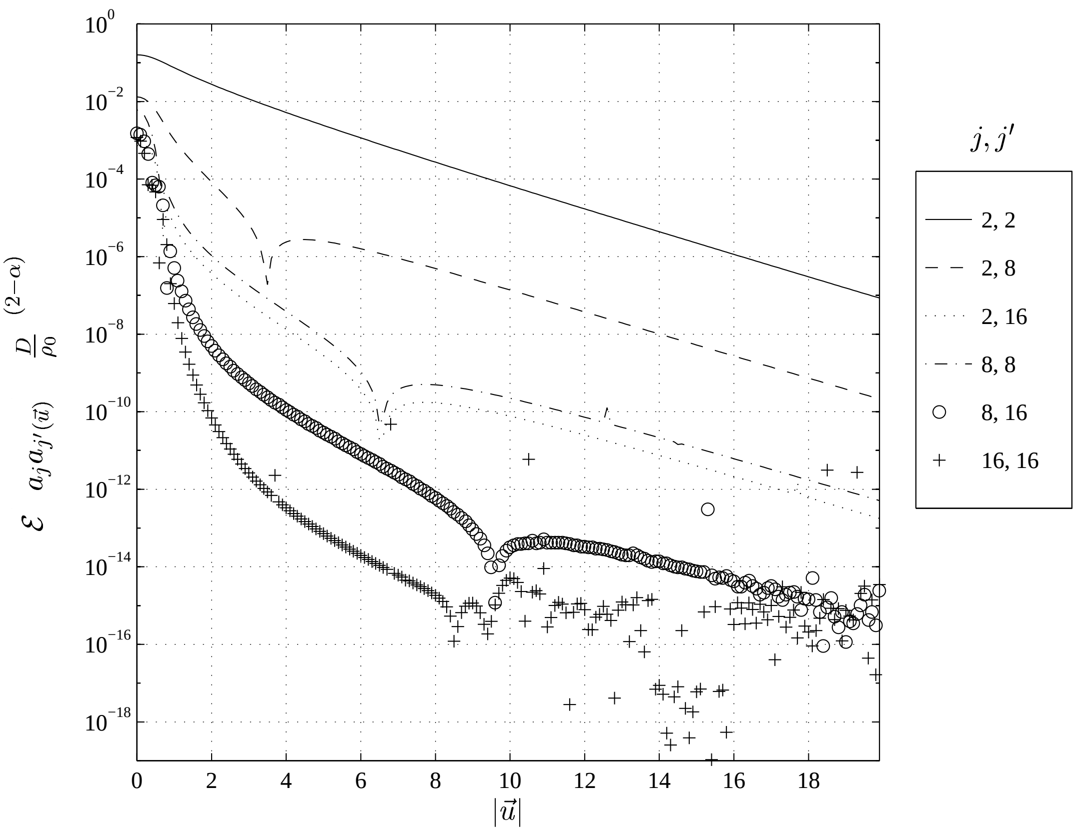
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



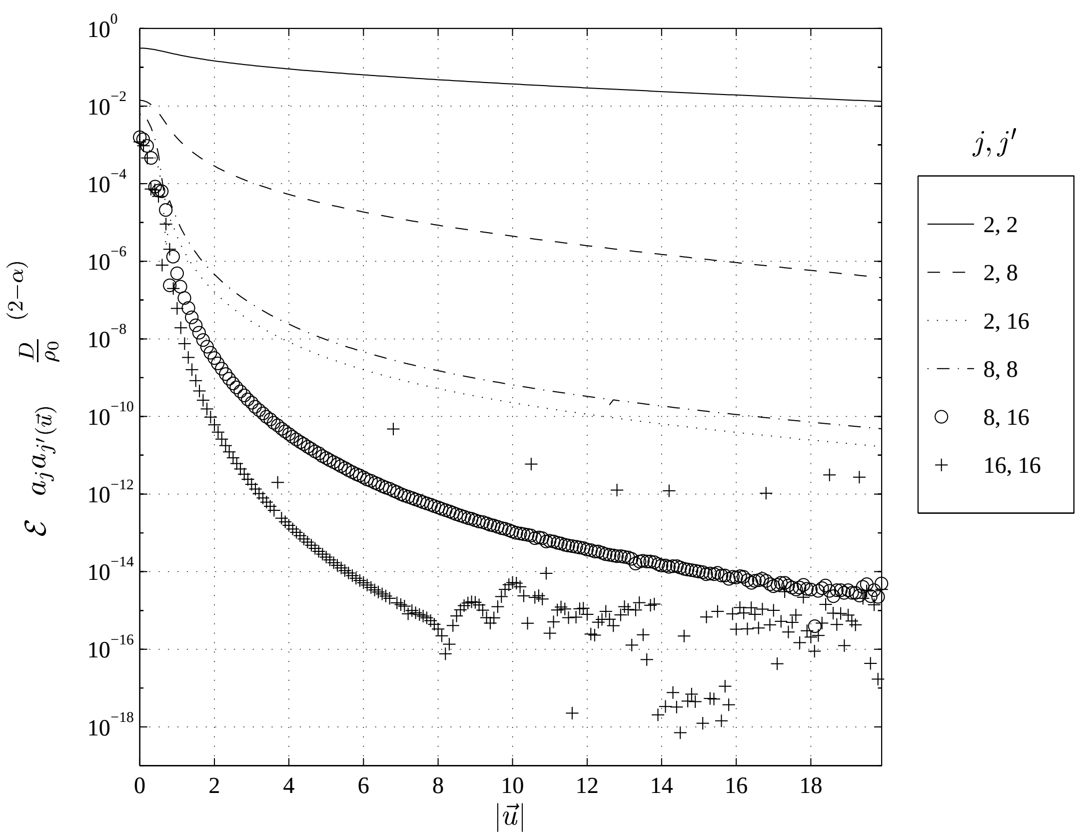
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



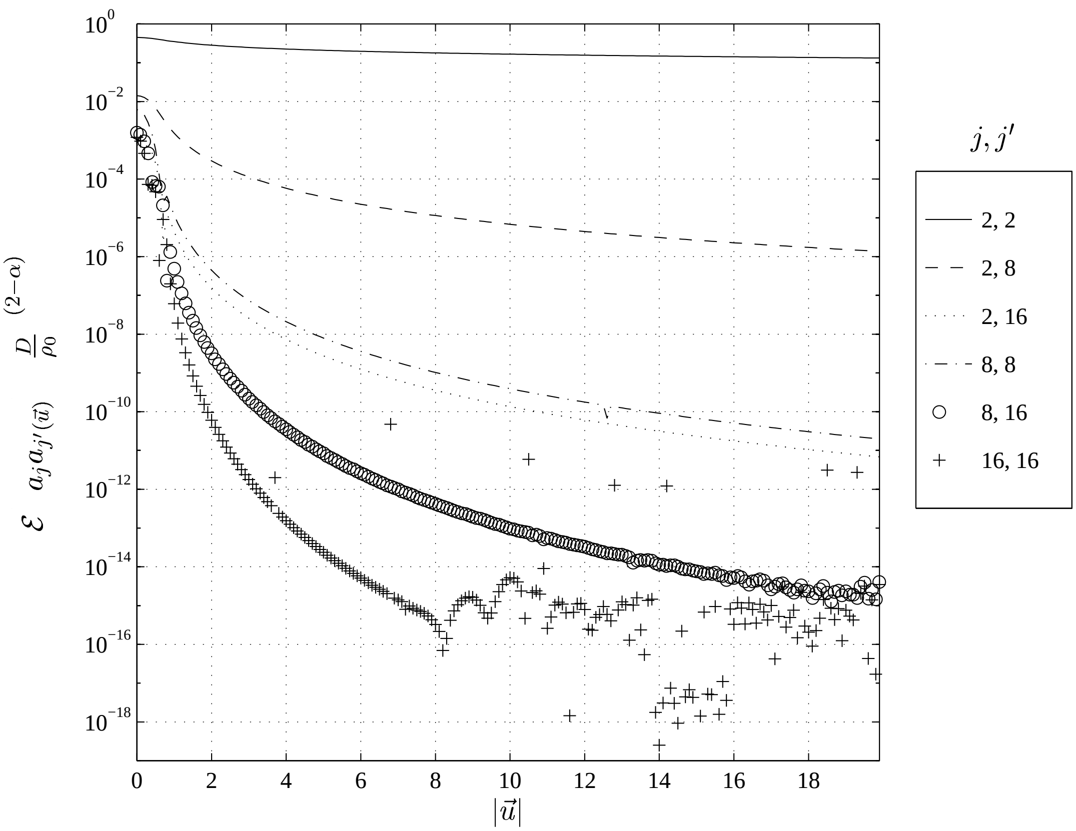
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .



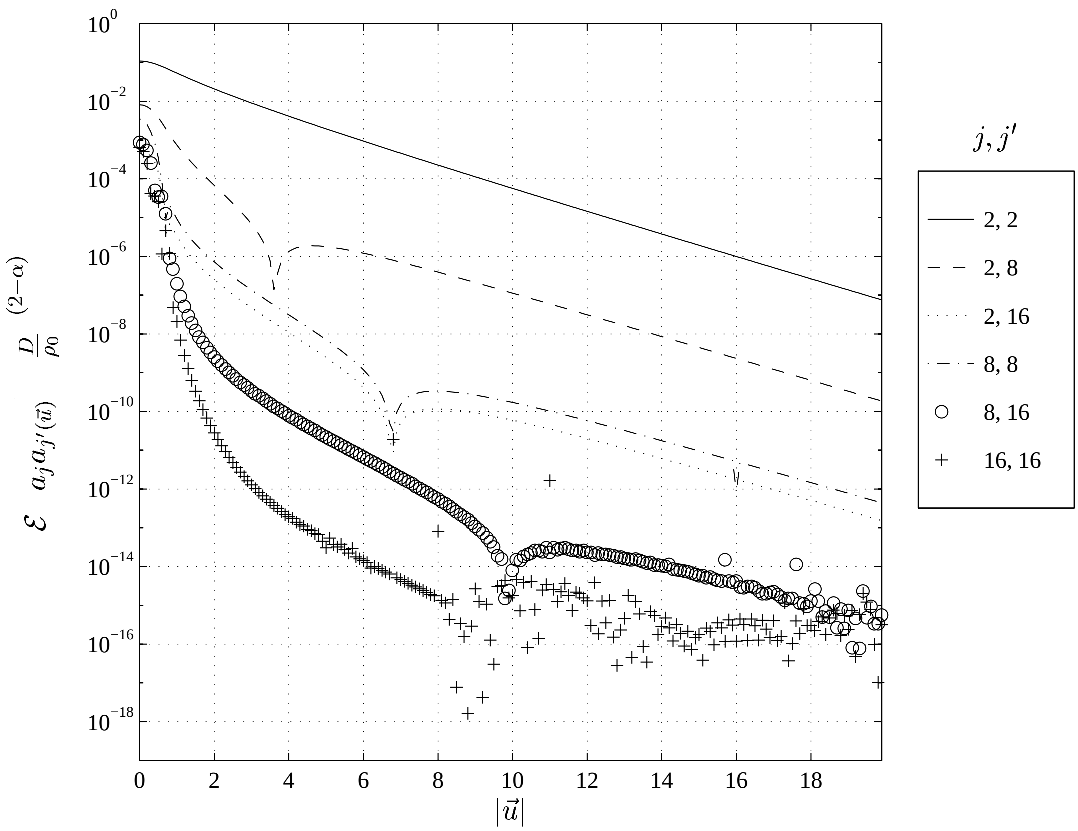
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



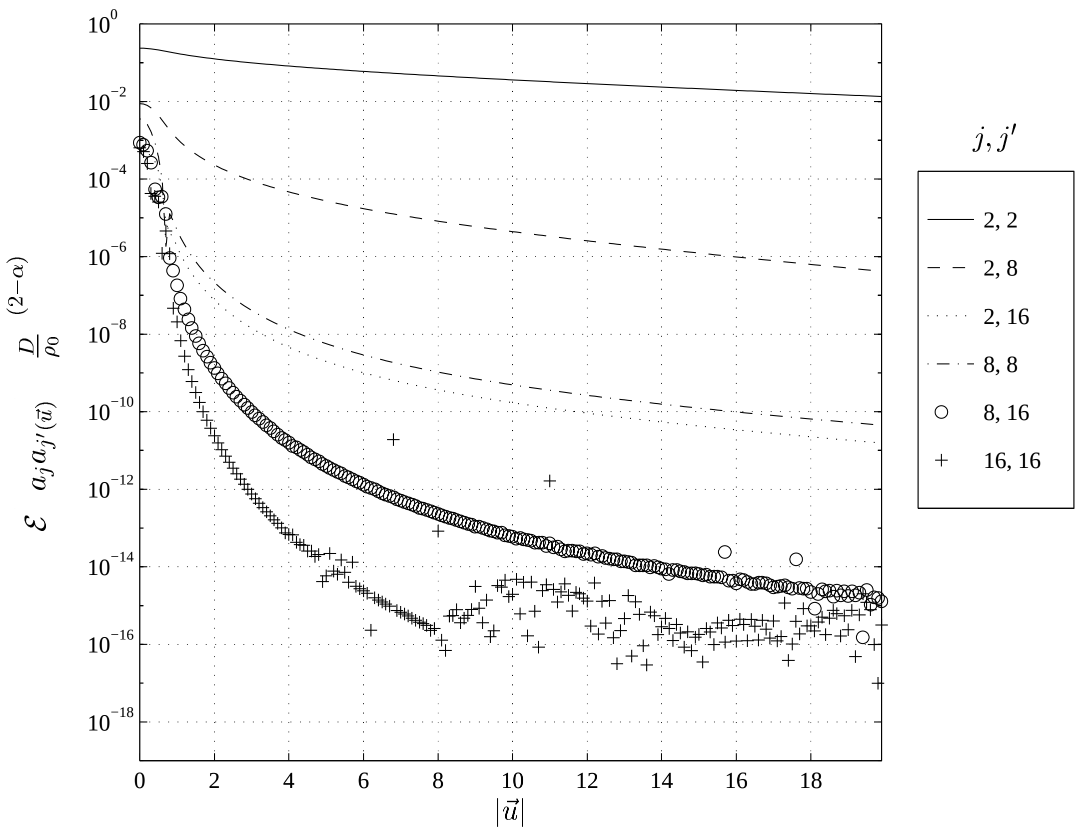
$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



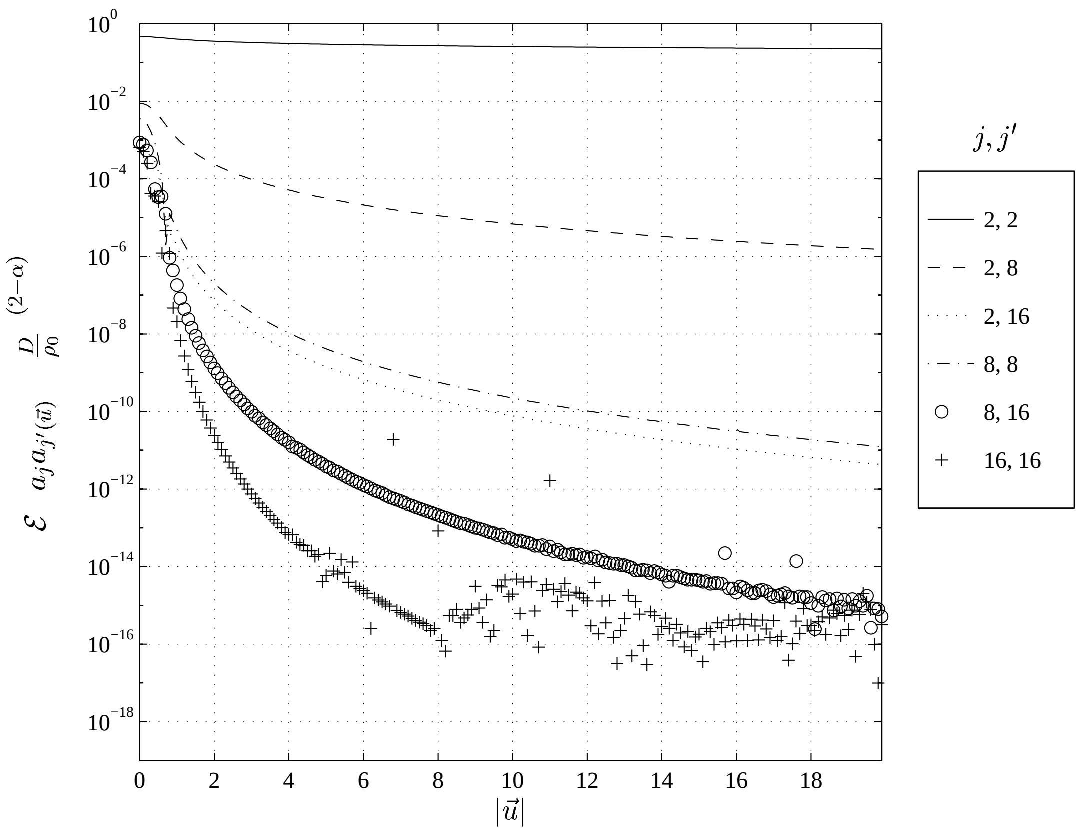
$\mathcal{E}\quad a_j a_{j'(\vec{u})}\quad \frac{D}{\rho_0}\ ^{(2-\alpha)}\qquad \theta_0\quad {}^{\circ}\ L_0/D\quad \infty\ \ \alpha\quad .$



$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D . α .



$\mathcal{E} \quad a_j a_{j'(\vec{u})} \quad \frac{D}{\rho_0}^{(2-\alpha)}$ θ_0 $\circ$ L_0/D . α .



$\mathcal{E}$ $a_j a_{j'(\vec{u})}$ $\frac{D}{\rho_0}$ $^{(2-\alpha)}$ θ_0 $^\circ$ L_0/D ∞ α .

$\mathcal{E} \quad a_j a_{j'}^*$

/

$\mathcal{E} \quad a_j^2$ /

$\vec{u}$ $\mathcal{E} \quad a_j a_{j'}^*$

arbitrary powers laws and finite outer scale

$\mathcal{E} \quad a_j a_{j'}^*$

	α	L_0/D
		∞
		∞
		∞

	α	L_0/D
		∞
		∞
		∞

$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha}$

α

L_0/D

$\mathcal{E} \quad a_1^2$

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.050213			-0.010201							-0.000399	
2		0.028387						-0.007338				
3			0.028387				-0.007338					
4	-0.010201			0.015906							-0.004724	
5					0.015906							
6						0.015906						-0.004724
7			-0.007338				0.009188					
8		-0.007338						0.009188				
9									0.009188			
10										0.009188		
11	-0.000399			-0.004724							0.005560	
12						-0.004724						0.005560

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.335123			-0.095702							0.002851	
2		0.195475						-0.027639				
3			0.195475				-0.027639					
4	-0.095702			0.049258							-0.010827	
5					0.049258							
6						0.049258						-0.010827
7			-0.027639				0.018558					
8		-0.027639						0.018558				
9									0.018558			
10										0.018558		
11	0.002851			-0.010827							0.008875	
12						-0.010827						0.008875

$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0}^{\;2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \quad .$

	1	2	3	4	5	6	7	8	9	10	11	12
1	3.649597			-0.134876							0.003566	
2		0.265769						-0.030644				
3			0.265769				-0.030644					
4	-0.134876			0.053825							-0.011296	
5					0.053825							
6						0.053825						-0.011296
7			-0.030644				0.019237					
8		-0.030644						0.019237				
9									0.019237			
10										0.019237		
11	0.003566			-0.011296							0.009049	
12						-0.011296						0.009049

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ ${}^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	57.178109			-0.189338							0.003921	
2		0.361194						-0.032035				
3			0.361194				-0.032035					
4	-0.189338			0.055904							-0.011465	
5					0.055904							
6						0.055904						-0.011465
7			-0.032035				0.019481					
8		-0.032035						0.019481				
9									0.019481			
10										0.019481		
11	0.003921			-0.011465							0.009109	
12						-0.011465						0.009109

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-0.198463							0.003926	
2		0.377013						-0.032053				
3			0.377013				-0.032053					
4	-0.198463			0.055931							-0.011467	
5					0.055931							
6						0.055931						-0.011467
7			-0.032053				0.019483					
8		-0.032053						0.019483				
9									0.019483			
10										0.019483		
11	0.003926			-0.011467							0.009110	
12						-0.011467						0.009110

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$ ∞

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.044059			-0.008941							-0.000297	
2		0.024553						-0.006328				
3			0.024553				-0.006328					
4	-0.008941			0.013540							-0.004008	
5					0.013540							
6						0.013540						-0.004008
7			-0.006328				0.007697					
8		-0.006328						0.007697				
9									0.007697			
10										0.007697		
11	-0.000297			-0.004008							0.004586	
12						-0.004008						0.004586

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.332826			-0.091201							0.002801	
2		0.184403						-0.025080				
3			0.184403				-0.025080					
4	-0.091201			0.044172							-0.009476	
5					0.044172							
6						0.044172						-0.009476
7			-0.025080				0.016044					
8		-0.025080						0.016044				
9									0.016044			
10										0.016044		
11	0.002801			-0.009476							0.007472	
12						-0.009476						0.007472

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	3.850011			-0.131720							0.003508	
2		0.256936						-0.027999				
3			0.256936				-0.027999					
4	-0.131720			0.048589							-0.009908	
5					0.048589							
6						0.048589						-0.009908
7			-0.027999				0.016667					
8		-0.027999						0.016667				
9									0.016667			
10										0.016667		
11	0.003508			-0.009908							0.007626	
12						-0.009908						0.007626

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	74.079729			-0.193801							0.003869	
2		0.365548						-0.029387				
3			0.365548				-0.029387					
4	-0.193801			0.050655							-0.010065	
5					0.050655							
6						0.050655						-0.010065
7			-0.029387				0.016892					
8		-0.029387						0.016892				
9									0.016892			
10										0.016892		
11	0.003869			-0.010065							0.007679	
12						-0.010065						0.007679

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-0.207083							0.003875	
2		0.388567						-0.029406				
3			0.388567				-0.029406					
4	-0.207083			0.050683							-0.010066	
5					0.050683							
6						0.050683						-0.010066
7			-0.029406				0.016894					
8		-0.029406						0.016894				
9									0.016894			
10										0.016894		
11	0.003875			-0.010066							0.007679	
12						-0.010066						0.007679

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$ ∞

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.032006			-0.006472							-0.000145	
2		0.017339						-0.004434				
3			0.017339				-0.004434					
4	-0.006472			0.009265							-0.002720	
5					0.009265							
6						0.009265						-0.002720
7			-0.004434				0.005100					
8		-0.004434						0.005100				
9									0.005100			
10										0.005100		
11	-0.000145			-0.002720							0.002947	
12						-0.002720						0.002947

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.256955			-0.078395							0.002499	
2		0.155630						-0.019529				
3			0.155630				-0.019529					
4	-0.078395			0.033655							-0.006855	
5					0.033655							
6						0.033655						-0.006855
7			-0.019529				0.011347					
8		-0.019529						0.011347				
9									0.011347			
10										0.011347		
11	0.002499			-0.006855							0.005007	
12						-0.006855						0.005007

$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad ^{2-\alpha} \quad \alpha \quad . \quad \frac{L_0}{D} \quad .$

	1	2	3	4	5	6	7	8	9	10	11	12
1	4.065806			-0.119397							0.003151	
2		0.228718						-0.022136				
3			0.228718				-0.022136					
4	-0.119397			0.037568							-0.007202	
5					0.037568							
6						0.037568						-0.007202
7			-0.022136				0.011843					
8		-0.022136						0.011843				
9									0.011843			
10										0.011843		
11	0.003151			-0.007202							0.005121	
12						-0.007202						0.005121

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	119.118810			-0.196789							0.003505	
2		0.363780						-0.023455				
3			0.363780				-0.023455					
4	-0.196789			0.039515							-0.007330	
5					0.039515							
6						0.039515						-0.007330
7			-0.023455				0.012023					
8		-0.023455						0.012023				
9									0.012023			
10										0.012023		
11	0.003505			-0.007330							0.005160	
12						-0.007330						0.005160

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-0.225207							0.003511	
2		0.413015						-0.023474				
3			0.413015				-0.023474					
4	-0.225207			0.039544							-0.007331	
5					0.039544							
6						0.039544						-0.007331
7			-0.023474				0.012025					
8		-0.023474						0.012025				
9									0.012025			
10										0.012025		
11	0.003511			-0.007331							0.005160	
12						-0.007331						0.005160

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$ ∞

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.016812			-0.003374							-0.000033	
2		0.008777						-0.002216				
3			0.008777				-0.002216					
4	-0.003374			0.004499							-0.001302	
5					0.004499							
6						0.004499						-0.001302
7			-0.002216				0.002374					
8		-0.002216						0.002374				
9									0.002374			
10										0.002374		
11	-0.000033			-0.001302							0.001317	
12						-0.001302						0.001317

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0} \quad {}^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \quad .$$

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.940439			-0.051899							0.001675	
2		0.100897						-0.011315				
3			0.100897				-0.011315					
4	-0.051899			0.019006							-0.003591	
5					0.019006							
6						0.019006						-0.003591
7			-0.011315				0.005786					
8		-0.011315						0.005786				
9									0.005786			
10										0.005786		
11	0.001675			-0.003591							0.002371	
12						-0.003591						0.002371

$$\mathcal{E} \quad a_j a_{j'} \quad \frac{D}{\rho_0}^{2-\alpha} \quad \alpha \quad \frac{11}{3} \quad \frac{L_0}{D} \quad .$$

	1	2	3	4	5	6	7	8	9	10	11	12
1	3.555503			-0.085591							0.002147	
2		0.160671						-0.013132				
3			0.160671				-0.013132					
4	-0.085591			0.021709							-0.003800	
5					0.021709							
6						0.021709						-0.003800
7			-0.013132				0.006081					
8		-0.013132						0.006081				
9									0.006081			
10										0.006081		
11	0.002147			-0.003800							0.002433	
12						-0.003800						0.002433

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ ${}^{2-\alpha}$ α $\frac{11}{3}$ $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	185.201300			-0.172147							0.002429	
2		0.311356						-0.014146				
3			0.311356				-0.014146					
4	-0.172147			0.023192							-0.003878	
5					0.023192							
6						0.023192						-0.003878
7			-0.014146				0.006190					
8		-0.014146						0.006190				
9									0.006190			
10										0.006190		
11	0.002429			-0.003878							0.002454	
12						-0.003878						0.002454

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α $\frac{11}{3}$ $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-0.251538							0.002434	
2		0.448879						-0.014164				
3			0.448879				-0.014164					
4	-0.251538			0.023218							-0.003879	
5					0.023218							
6						0.023218						-0.003879
7			-0.014164				0.006191					
8		-0.014164						0.006191				
9									0.006191			
10										0.006191		
11	0.002434			-0.003879							0.002454	
12						-0.003879						0.002454

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α $\frac{11}{3}$ $\frac{L_0}{D}$ ∞

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.009744			-0.001947							-0.000008	
2		0.004996						-0.001252				
3			0.004996				-0.001252					
4	-0.001947			0.002509							-0.000720	
5					0.002509							
6						0.002509						-0.000720
7			-0.001252				0.001296					
8		-0.001252						0.001296				
9									0.001296			
10										0.001296		
11	-0.000008			-0.000720							0.000705	
12						-0.000720						0.000705

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.652068			-0.033878							0.001087	
2		0.065262						-0.006906				
3			0.065262				-0.006906					
4	-0.033878			0.011467							-0.002082	
5					0.011467							
6						0.011467						-0.002082
7			-0.006906				0.003313					
8		-0.006906						0.003313				
9									0.003313			
10										0.003313		
11	0.001087			-0.002082							0.001308	
12						-0.002082						0.001308

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	2.670352			-0.058364							0.001409	
2		0.108616						-0.008125				
3			0.108616				-0.008125					
4	-0.058364			0.013272							-0.002211	
5					0.013272							
6						0.013272						-0.002211
7			-0.008125				0.003495					
8		-0.008125						0.003495				
9									0.003495			
10										0.003495		
11	0.001409			-0.002211							0.001344	
12						-0.002211						0.001344

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	186.340090			-0.132721							0.001612	
2		0.237945						-0.008843				
3			0.237945				-0.008843					
4	-0.132721			0.014319							-0.002261	
5					0.014319							
6						0.014319						-0.002261
7			-0.008843				0.003564					
8		-0.008843						0.003564				
9									0.003564			
10										0.003564		
11	0.001612			-0.002261							0.001356	
12						-0.002261						0.001356

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-0.265780							0.001616	
2		0.468420						-0.008857				
3			0.468420				-0.008857					
4	-0.265780			0.014339							-0.002261	
5					0.014339							
6						0.014339						-0.002261
7			-0.008857				0.003565					
8		-0.008857						0.003565				
9									0.003565			
10										0.003565		
11	0.001616			-0.002261							0.001356	
12						-0.002261						0.001356

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$ ∞

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.004732			-0.000942							-0.000000	
2		0.002394						-0.000596				
3			0.002394				-0.000596					
4	-0.000942			0.001184							-0.000338	
5					0.001184							
6						0.001184						-0.000338
7			-0.000596				0.000602					
8		-0.000596						0.000602				
9									0.000602			
10										0.000602		
11	-0.000000			-0.000338							0.000322	
12						-0.000338						0.000322

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.362566			-0.018012							0.000573	
2		0.034478						-0.003491				
3			0.034478				-0.003491					
4	-0.018012			0.005749							-0.001012	
5					0.005749							
6						0.005749						-0.001012
7			-0.003491				0.001596					
8		-0.003491						0.001596				
9									0.001596			
10										0.001596		
11	0.000573			-0.001012							0.000612	
12						-0.001012						0.000612

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.577670			-0.032118							0.000749	
2		0.059419						-0.004152				
3			0.059419				-0.004152					
4	-0.032118			0.006726							-0.001078	
5					0.006726							
6						0.006726						-0.001078
7			-0.004152				0.001689					
8		-0.004152						0.001689				
9									0.001689			
10										0.001689		
11	0.000749			-0.001078							0.000630	
12						-0.001078						0.000630

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1	137.306270			-0.080958							0.000866	
2		0.144317						-0.004560				
3			0.144317				-0.004560					
4	-0.080958			0.007318							-0.001104	
5					0.007318							
6						0.007318						-0.001104
7			-0.004560				0.001725					
8		-0.004560						0.001725				
9									0.001725			
10										0.001725		
11	0.000866			-0.001104							0.000636	
12						-0.001104						0.000636

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}$ $^{2-\alpha}$ α . $\frac{L_0}{D}$.

	1	2	3	4	5	6	7	8	9	10	11	12
1				-2.527445							0.000869	
2								-0.004569				
3							-0.004569					
4	-2.527445			0.007331							-0.001104	
5					0.007331							
6						0.007331						-0.001104
7			-0.004569				0.001725					
8		-0.004569						0.001725				
9									0.001725			
10										0.001725		
11	0.000869			-0.001104							0.000636	
12						-0.001104						0.000636

$\mathcal{E}$ $a_j a_{j'}$ $\frac{D}{\rho_0}^{2-\alpha}$ α . $\frac{L_0}{D}$ ∞

...

```
(*------------------------------------------------------------
This Mathematica code helps determine the second moment for the
slope structure function estimator.

x2-x1=p
x4-x3=p

case1: x1=x3 and x2=x4 and delta_time=0
case2: x2=x3 and delta_time=0
case3: x4=x1 and delta_time=0
case4: x1,x2,x3,and x4 are four unique location (no overlap)

Toby D. Reeves, July-Sept 1996
*)
(*------------------------------------------------------------
*)
Remove["Global`*"]
(*------------------------------------------------------------
Define the Expected Value Operator
*)
EV[a_+b_]:=EV[a]+EV[b]
EV[a_Integer b_]:=a EV[b]
(*------------------------------------------------------------
 Define a Wrapper zmgrv[ ] for Zero Mean Gaussian Random Variable,
        then define rules that apply to zmgrv.
*)
Format[zmgrv[x_]]:=x
EV[u1_zmgrv u2_zmgrv u3_zmgrv u4_zmgrv]:= (
        EV[u1 u2] EV[u3 u4] +
        EV[u1 u3] EV[u2 u4] +
        EV[u1 u4] EV[u2 u3]
        )
EV[u1_zmgrv u1_zmgrv u3_zmgrv u4_zmgrv]:= (
        EV[u1 u2] EV[u3 u4] +
```

```
        EV[u1 u3] EV[u2 u4] +
        EV[u1 u4] EV[u2 u3]
        )/.u2->u1
EV[u1_zmgrv u1_zmgrv u3_zmgrv u3_zmgrv]:= (
        EV[u1 u2] EV[u3 u4] +
        EV[u1 u3] EV[u2 u4] +
        EV[u1 u4] EV[u2 u3]
        )/.{u2->u1,u4->u3}
EV[u1_zmgrv u1_zmgrv u1_zmgrv u4_zmgrv]:= (
        EV[u1 u2] EV[u3 u4] +
        EV[u1 u3] EV[u2 u4] +
        EV[u1 u4] EV[u2 u3]
        )/.{u2->u1,u3->u1}
EV[u1_zmgrv u1_zmgrv u1_zmgrv u1_zmgrv]:= (
        EV[u1 u2] EV[u3 u4] +
        EV[u1 u3] EV[u2 u4] +
        EV[u1 u4] EV[u2 u3]
        )/.{u2->u1,u3->u1,u4->u1}
(*---------------------------------------------------------------
Properties of the Noise
*)
noiseRules= {
        EV[zmgrv[n[a_]] zmgrv[n[a_]]]->sig^2,
        EV[zmgrv[n[a_]] zmgrv[n[b_]]]->0,
        EV[zmgrv[n[a_]]]->0,
        EV[zmgrv[n[a_]] zmgrv[s[x_]]]->0
};
(*---------------------------------------------------------------
        Apply these rules to my problem of calculating the expectation
        within the Second Moment formula
*)
f1= (a-b)^2 (c-d)^2 /.
        {a->zmgrv[s[x1]]+zmgrv[n[x1]],
         b->zmgrv[s[x2]]+zmgrv[n[x2]],
         c->zmgrv[s[x3]]+zmgrv[n[x3]],
         d->zmgrv[s[x4]]+zmgrv[n[x4]]
        };
BigMess=EV[Expand[f1]];
convertRule={
        EV[zmgrv[s[x_]]^2]      ->G[0],

        EV[zmgrv[s[x1]] zmgrv[s[x2]]]   ->G[x2-x1],
        EV[zmgrv[s[x3]] zmgrv[s[x4]]]   ->G[x4-x3],

        EV[zmgrv[s[x1]] zmgrv[s[x3]]]   ->G[x3-x1],
```

```
        EV[zmgrv[s[x2]] zmgrv[s[x4]]]   ->G[x4-x2],

        EV[zmgrv[s[x1]] zmgrv[s[x4]]]   ->G[x4-x1],
        EV[zmgrv[s[x2]] zmgrv[s[x3]]]   ->G[x3-x2]
};

(* apply geometry and noiseRules for each case *)
case1Rule={x1->x3,x2->x4};
case1a=(BigMess/.case1Rule)/.noiseRules;

case2Rule={x2->x3};
case2a=(BigMess/.case2Rule)/.noiseRules;

case3Rule={x1->x4};
case3a=(BigMess/.case3Rule)/.noiseRules;

case4Rule={};
case4a=(BigMess/.case4Rule)/.noiseRules;

(* write in Gamma notation *)
case1b=case1a/.convertRule;
case2b=case2a/.convertRule;
case3b=case3a/.convertRule;
case4b=case4a/.convertRule;

(* use geometry to simplify more *)
geoRule={x2-x1->p,x4-x3->p,x4-x2->x3-x1} (*valid all cases*)
case1c=ExpandAll[case1b/.geoRule];
case2c=ExpandAll[(case2b/.geoRule)/.{x3-x1->p,x4-x1->2p}];
case3c=ExpandAll[(case3b/.geoRule)/.{x3-x1->-p,x3-x2->-2p}];
case4c=ExpandAll[case4b/.geoRule];

(* We know that for our case G[p]=G[-p] *)
q=case3c
case3c=case3c/.{-2p -> 2p, -p -> p};

(*-----------------------------------------------------------
Drum roll please!!
*)
sout=OpenWrite["sf5n.txt",FormatType->OutputForm,PageWidth->75]

Write[sout,"BigMess= ",ExpandAll[BigMess]]

Write[sout,"case1a= ",case1a]
Write[sout,"case2a= ",case2a]
```

```
Write[sout,"case3a= ",case3a]
Write[sout,"case4a= ",case4a]

Write[sout,"case1b= ",case1b]
Write[sout,"case2b= ",case2b]
Write[sout,"case3b= ",case3b]
Write[sout,"case4b= ",case4b]

Write[sout,"case1c= ",case1c]
Write[sout,"case2c= ",case2c]
Write[sout,"case3c= ",case3c]
Write[sout,"case4c= ",case4c]

Close[sout]

                            2
BigMess= 2 EV[n[x1] n[x3]]  - 4 EV[n[x1] n[x3]] EV[n[x2] n[x3]] +

                        2            2           2
>     2 EV[n[x2] n[x3]]  + EV[n[x1] ] EV[n[x3] ] -

                                  2            2           2
>     2 EV[n[x1] n[x2]] EV[n[x3] ] + EV[n[x2] ] EV[n[x3] ] -

>     4 EV[n[x1] n[x3]] EV[n[x1] n[x4]] +

                                                             2
>     4 EV[n[x2] n[x3]] EV[n[x1] n[x4]] + 2 EV[n[x1] n[x4]]  +

>     4 EV[n[x1] n[x3]] EV[n[x2] n[x4]] -

>     4 EV[n[x2] n[x3]] EV[n[x2] n[x4]] -

                                                             2
>     4 EV[n[x1] n[x4]] EV[n[x2] n[x4]] + 2 EV[n[x2] n[x4]]  -

                2
>     2 EV[n[x1] ] EV[n[x3] n[x4]] + 4 EV[n[x1] n[x2]] EV[n[x3] n[x4]] -

                2                              2           2
>     2 EV[n[x2] ] EV[n[x3] n[x4]] + EV[n[x1] ] EV[n[x4] ] -

                                  2            2           2
>     2 EV[n[x1] n[x2]] EV[n[x4] ] + EV[n[x2] ] EV[n[x4] ] +

                2
```

```
>    2 EV[n[x3] ] EV[n[x1] s[x1]] - 4 EV[n[x3] n[x4]] EV[n[x1] s[x1]] +

               2                              2
>    2 EV[n[x4] ] EV[n[x1] s[x1]] - 2 EV[n[x3] ] EV[n[x2] s[x1]] +

                                                   2
>    4 EV[n[x3] n[x4]] EV[n[x2] s[x1]] - 2 EV[n[x4] ] EV[n[x2] s[x1]] +

>    4 EV[n[x1] n[x3]] EV[n[x3] s[x1]] -

>    4 EV[n[x2] n[x3]] EV[n[x3] s[x1]] -

>    4 EV[n[x1] n[x4]] EV[n[x3] s[x1]] +

                                                          2
>    4 EV[n[x2] n[x4]] EV[n[x3] s[x1]] + 2 EV[n[x3] s[x1]]  -

>    4 EV[n[x1] n[x3]] EV[n[x4] s[x1]] +

>    4 EV[n[x2] n[x3]] EV[n[x4] s[x1]] +

>    4 EV[n[x1] n[x4]] EV[n[x4] s[x1]] -

>    4 EV[n[x2] n[x4]] EV[n[x4] s[x1]] -

                                                          2
>    4 EV[n[x3] s[x1]] EV[n[x4] s[x1]] + 2 EV[n[x4] s[x1]]  +

             2          2                              2
>    EV[n[x3] ] EV[s[x1] ] - 2 EV[n[x3] n[x4]] EV[s[x1] ] +

             2          2              2
>    EV[n[x4] ] EV[s[x1] ] - 2 EV[n[x3] ] EV[n[x1] s[x2]] +

                                                   2
>    4 EV[n[x3] n[x4]] EV[n[x1] s[x2]] - 2 EV[n[x4] ] EV[n[x1] s[x2]] +

               2
>    2 EV[n[x3] ] EV[n[x2] s[x2]] - 4 EV[n[x3] n[x4]] EV[n[x2] s[x2]] +

               2
>    2 EV[n[x4] ] EV[n[x2] s[x2]] - 4 EV[n[x1] n[x3]] EV[n[x3] s[x2]] +

>    4 EV[n[x2] n[x3]] EV[n[x3] s[x2]] +
```

```
>    4 EV[n[x1] n[x4]] EV[n[x3] s[x2]] -

>    4 EV[n[x2] n[x4]] EV[n[x3] s[x2]] -

>    4 EV[n[x3] s[x1]] EV[n[x3] s[x2]] +

                                                          2
>    4 EV[n[x4] s[x1]] EV[n[x3] s[x2]] + 2 EV[n[x3] s[x2]]  +

>    4 EV[n[x1] n[x3]] EV[n[x4] s[x2]] -

>    4 EV[n[x2] n[x3]] EV[n[x4] s[x2]] -

>    4 EV[n[x1] n[x4]] EV[n[x4] s[x2]] +

>    4 EV[n[x2] n[x4]] EV[n[x4] s[x2]] +

>    4 EV[n[x3] s[x1]] EV[n[x4] s[x2]] -

>    4 EV[n[x4] s[x1]] EV[n[x4] s[x2]] -

                                                          2
>    4 EV[n[x3] s[x2]] EV[n[x4] s[x2]] + 2 EV[n[x4] s[x2]]  -

               2
>    2 EV[n[x3] ] EV[s[x1] s[x2]] + 4 EV[n[x3] n[x4]] EV[s[x1] s[x2]] -

               2                            2          2
>    2 EV[n[x4] ] EV[s[x1] s[x2]] + EV[n[x3] ] EV[s[x2] ] -

                               2             2          2
>    2 EV[n[x3] n[x4]] EV[s[x2] ] + EV[n[x4] ] EV[s[x2] ] +

>    4 EV[n[x1] n[x3]] EV[n[x1] s[x3]] -

>    4 EV[n[x2] n[x3]] EV[n[x1] s[x3]] -

>    4 EV[n[x1] n[x4]] EV[n[x1] s[x3]] +

>    4 EV[n[x2] n[x4]] EV[n[x1] s[x3]] +

>    4 EV[n[x3] s[x1]] EV[n[x1] s[x3]] -

>    4 EV[n[x4] s[x1]] EV[n[x1] s[x3]] -
```

```
>    4 EV[n[x3] s[x2]] EV[n[x1] s[x3]] +

                                                     2
>    4 EV[n[x4] s[x2]] EV[n[x1] s[x3]] + 2 EV[n[x1] s[x3]]  -

>    4 EV[n[x1] n[x3]] EV[n[x2] s[x3]] +

>    4 EV[n[x2] n[x3]] EV[n[x2] s[x3]] +

>    4 EV[n[x1] n[x4]] EV[n[x2] s[x3]] -

>    4 EV[n[x2] n[x4]] EV[n[x2] s[x3]] -

>    4 EV[n[x3] s[x1]] EV[n[x2] s[x3]] +

>    4 EV[n[x4] s[x1]] EV[n[x2] s[x3]] +

>    4 EV[n[x3] s[x2]] EV[n[x2] s[x3]] -

>    4 EV[n[x4] s[x2]] EV[n[x2] s[x3]] -

                                                     2
>    4 EV[n[x1] s[x3]] EV[n[x2] s[x3]] + 2 EV[n[x2] s[x3]]  +

          2
>    2 EV[n[x1] ] EV[n[x3] s[x3]] - 4 EV[n[x1] n[x2]] EV[n[x3] s[x3]] +

          2
>    2 EV[n[x2] ] EV[n[x3] s[x3]] + 4 EV[n[x1] s[x1]] EV[n[x3] s[x3]] -

                                              2
>    4 EV[n[x2] s[x1]] EV[n[x3] s[x3]] + 2 EV[s[x1] ] EV[n[x3] s[x3]] -

>    4 EV[n[x1] s[x2]] EV[n[x3] s[x3]] +

>    4 EV[n[x2] s[x2]] EV[n[x3] s[x3]] -

                                              2
>    4 EV[s[x1] s[x2]] EV[n[x3] s[x3]] + 2 EV[s[x2] ] EV[n[x3] s[x3]] -

          2
>    2 EV[n[x1] ] EV[n[x4] s[x3]] + 4 EV[n[x1] n[x2]] EV[n[x4] s[x3]] -

          2
>    2 EV[n[x2] ] EV[n[x4] s[x3]] - 4 EV[n[x1] s[x1]] EV[n[x4] s[x3]] +
```

```
                                                      2
>    4 EV[n[x2] s[x1]] EV[n[x4] s[x3]] - 2 EV[s[x1] ] EV[n[x4] s[x3]] +

>    4 EV[n[x1] s[x2]] EV[n[x4] s[x3]] -

>    4 EV[n[x2] s[x2]] EV[n[x4] s[x3]] +

                                                      2
>    4 EV[s[x1] s[x2]] EV[n[x4] s[x3]] - 2 EV[s[x2] ] EV[n[x4] s[x3]] +

>    4 EV[n[x1] n[x3]] EV[s[x1] s[x3]] -

>    4 EV[n[x2] n[x3]] EV[s[x1] s[x3]] -

>    4 EV[n[x1] n[x4]] EV[s[x1] s[x3]] +

>    4 EV[n[x2] n[x4]] EV[s[x1] s[x3]] +

>    4 EV[n[x3] s[x1]] EV[s[x1] s[x3]] -

>    4 EV[n[x4] s[x1]] EV[s[x1] s[x3]] -

>    4 EV[n[x3] s[x2]] EV[s[x1] s[x3]] +

>    4 EV[n[x4] s[x2]] EV[s[x1] s[x3]] +

>    4 EV[n[x1] s[x3]] EV[s[x1] s[x3]] -

                                                             2
>    4 EV[n[x2] s[x3]] EV[s[x1] s[x3]] + 2 EV[s[x1] s[x3]]  -

>    4 EV[n[x1] n[x3]] EV[s[x2] s[x3]] +

>    4 EV[n[x2] n[x3]] EV[s[x2] s[x3]] +

>    4 EV[n[x1] n[x4]] EV[s[x2] s[x3]] -

>    4 EV[n[x2] n[x4]] EV[s[x2] s[x3]] -

>    4 EV[n[x3] s[x1]] EV[s[x2] s[x3]] +

>    4 EV[n[x4] s[x1]] EV[s[x2] s[x3]] +

>    4 EV[n[x3] s[x2]] EV[s[x2] s[x3]] -
```

```
>     4 EV[n[x4] s[x2]] EV[s[x2] s[x3]] -

>     4 EV[n[x1] s[x3]] EV[s[x2] s[x3]] +

>     4 EV[n[x2] s[x3]] EV[s[x2] s[x3]] -

                                                           2
>     4 EV[s[x1] s[x3]] EV[s[x2] s[x3]] + 2 EV[s[x2] s[x3]]  +

              2          2                              2
>     EV[n[x1] ] EV[s[x3] ] - 2 EV[n[x1] n[x2]] EV[s[x3] ] +

              2          2                              2
>     EV[n[x2] ] EV[s[x3] ] + 2 EV[n[x1] s[x1]] EV[s[x3] ] -

                                2            2          2
>     2 EV[n[x2] s[x1]] EV[s[x3] ] + EV[s[x1] ] EV[s[x3] ] -

                                2                              2
>     2 EV[n[x1] s[x2]] EV[s[x3] ] + 2 EV[n[x2] s[x2]] EV[s[x3] ] -

                                2            2          2
>     2 EV[s[x1] s[x2]] EV[s[x3] ] + EV[s[x2] ] EV[s[x3] ] -

>     4 EV[n[x1] n[x3]] EV[n[x1] s[x4]] +

>     4 EV[n[x2] n[x3]] EV[n[x1] s[x4]] +

>     4 EV[n[x1] n[x4]] EV[n[x1] s[x4]] -

>     4 EV[n[x2] n[x4]] EV[n[x1] s[x4]] -

>     4 EV[n[x3] s[x1]] EV[n[x1] s[x4]] +

>     4 EV[n[x4] s[x1]] EV[n[x1] s[x4]] +

>     4 EV[n[x3] s[x2]] EV[n[x1] s[x4]] -

>     4 EV[n[x4] s[x2]] EV[n[x1] s[x4]] -

>     4 EV[n[x1] s[x3]] EV[n[x1] s[x4]] +

>     4 EV[n[x2] s[x3]] EV[n[x1] s[x4]] -
```

```
>     4 EV[s[x1] s[x3]] EV[n[x1] s[x4]] +

                                                        2
>     4 EV[s[x2] s[x3]] EV[n[x1] s[x4]] + 2 EV[n[x1] s[x4]]  +

>     4 EV[n[x1] n[x3]] EV[n[x2] s[x4]] -

>     4 EV[n[x2] n[x3]] EV[n[x2] s[x4]] -

>     4 EV[n[x1] n[x4]] EV[n[x2] s[x4]] +

>     4 EV[n[x2] n[x4]] EV[n[x2] s[x4]] +

>     4 EV[n[x3] s[x1]] EV[n[x2] s[x4]] -

>     4 EV[n[x4] s[x1]] EV[n[x2] s[x4]] -

>     4 EV[n[x3] s[x2]] EV[n[x2] s[x4]] +

>     4 EV[n[x4] s[x2]] EV[n[x2] s[x4]] +

>     4 EV[n[x1] s[x3]] EV[n[x2] s[x4]] -

>     4 EV[n[x2] s[x3]] EV[n[x2] s[x4]] +

>     4 EV[s[x1] s[x3]] EV[n[x2] s[x4]] -

>     4 EV[s[x2] s[x3]] EV[n[x2] s[x4]] -

                                                        2
>     4 EV[n[x1] s[x4]] EV[n[x2] s[x4]] + 2 EV[n[x2] s[x4]]  -

                2
>     2 EV[n[x1] ] EV[n[x3] s[x4]] + 4 EV[n[x1] n[x2]] EV[n[x3] s[x4]] -

                2
>     2 EV[n[x2] ] EV[n[x3] s[x4]] - 4 EV[n[x1] s[x1]] EV[n[x3] s[x4]] +

                                                 2
>     4 EV[n[x2] s[x1]] EV[n[x3] s[x4]] - 2 EV[s[x1] ] EV[n[x3] s[x4]] +

>     4 EV[n[x1] s[x2]] EV[n[x3] s[x4]] -

>     4 EV[n[x2] s[x2]] EV[n[x3] s[x4]] +
```

```
                                                   2
>    4 EV[s[x1] s[x2]] EV[n[x3] s[x4]] - 2 EV[s[x2] ] EV[n[x3] s[x4]] +

               2
>    2 EV[n[x1] ] EV[n[x4] s[x4]] - 4 EV[n[x1] n[x2]] EV[n[x4] s[x4]] +

               2
>    2 EV[n[x2] ] EV[n[x4] s[x4]] + 4 EV[n[x1] s[x1]] EV[n[x4] s[x4]] -

                                                   2
>    4 EV[n[x2] s[x1]] EV[n[x4] s[x4]] + 2 EV[s[x1] ] EV[n[x4] s[x4]] -

>    4 EV[n[x1] s[x2]] EV[n[x4] s[x4]] +

>    4 EV[n[x2] s[x2]] EV[n[x4] s[x4]] -

                                                   2
>    4 EV[s[x1] s[x2]] EV[n[x4] s[x4]] + 2 EV[s[x2] ] EV[n[x4] s[x4]] -

>    4 EV[n[x1] n[x3]] EV[s[x1] s[x4]] +

>    4 EV[n[x2] n[x3]] EV[s[x1] s[x4]] +

>    4 EV[n[x1] n[x4]] EV[s[x1] s[x4]] -

>    4 EV[n[x2] n[x4]] EV[s[x1] s[x4]] -

>    4 EV[n[x3] s[x1]] EV[s[x1] s[x4]] +

>    4 EV[n[x4] s[x1]] EV[s[x1] s[x4]] +

>    4 EV[n[x3] s[x2]] EV[s[x1] s[x4]] -

>    4 EV[n[x4] s[x2]] EV[s[x1] s[x4]] -

>    4 EV[n[x1] s[x3]] EV[s[x1] s[x4]] +

>    4 EV[n[x2] s[x3]] EV[s[x1] s[x4]] -

>    4 EV[s[x1] s[x3]] EV[s[x1] s[x4]] +

>    4 EV[s[x2] s[x3]] EV[s[x1] s[x4]] +

>    4 EV[n[x1] s[x4]] EV[s[x1] s[x4]] -
```

```
                                                          2
>    4 EV[n[x2] s[x4]] EV[s[x1] s[x4]] + 2 EV[s[x1] s[x4]]  +

>    4 EV[n[x1] n[x3]] EV[s[x2] s[x4]] -

>    4 EV[n[x2] n[x3]] EV[s[x2] s[x4]] -

>    4 EV[n[x1] n[x4]] EV[s[x2] s[x4]] +

>    4 EV[n[x2] n[x4]] EV[s[x2] s[x4]] +

>    4 EV[n[x3] s[x1]] EV[s[x2] s[x4]] -

>    4 EV[n[x4] s[x1]] EV[s[x2] s[x4]] -

>    4 EV[n[x3] s[x2]] EV[s[x2] s[x4]] +

>    4 EV[n[x4] s[x2]] EV[s[x2] s[x4]] +

>    4 EV[n[x1] s[x3]] EV[s[x2] s[x4]] -

>    4 EV[n[x2] s[x3]] EV[s[x2] s[x4]] +

>    4 EV[s[x1] s[x3]] EV[s[x2] s[x4]] -

>    4 EV[s[x2] s[x3]] EV[s[x2] s[x4]] -

>    4 EV[n[x1] s[x4]] EV[s[x2] s[x4]] +

>    4 EV[n[x2] s[x4]] EV[s[x2] s[x4]] -

                                                          2
>    4 EV[s[x1] s[x4]] EV[s[x2] s[x4]] + 2 EV[s[x2] s[x4]]  -

               2
>    2 EV[n[x1] ] EV[s[x3] s[x4]] + 4 EV[n[x1] n[x2]] EV[s[x3] s[x4]] -

               2
>    2 EV[n[x2] ] EV[s[x3] s[x4]] - 4 EV[n[x1] s[x1]] EV[s[x3] s[x4]] +

                                                    2
>    4 EV[n[x2] s[x1]] EV[s[x3] s[x4]] - 2 EV[s[x1] ] EV[s[x3] s[x4]] +

>    4 EV[n[x1] s[x2]] EV[s[x3] s[x4]] -
```

```
>    4 EV[n[x2] s[x2]] EV[s[x3] s[x4]] +

                                                   2
>    4 EV[s[x1] s[x2]] EV[s[x3] s[x4]] - 2 EV[s[x2] ] EV[s[x3] s[x4]] +

             2          2                              2
>    EV[n[x1] ] EV[s[x4] ] - 2 EV[n[x1] n[x2]] EV[s[x4] ] +

             2          2                              2
>    EV[n[x2] ] EV[s[x4] ] + 2 EV[n[x1] s[x1]] EV[s[x4] ] -

                               2            2          2
>    2 EV[n[x2] s[x1]] EV[s[x4] ] + EV[s[x1] ] EV[s[x4] ] -

                               2                              2
>    2 EV[n[x1] s[x2]] EV[s[x4] ] + 2 EV[n[x2] s[x2]] EV[s[x4] ] -

                               2            2          2
>    2 EV[s[x1] s[x2]] EV[s[x4] ] + EV[s[x2] ] EV[s[x4] ]
              4         2         2              2 2
case1a= 12 sig  + 12 sig  EV[s[x3] ] + 3 EV[s[x3] ]  -

           2                              2
>    24 sig  EV[s[x3] s[x4]] - 12 EV[s[x3] ] EV[s[x3] s[x4]] +

                      2         2         2              2          2
>    4 EV[s[x3] s[x4]]  + 12 sig  EV[s[x4] ] + 2 EV[s[x3] ] EV[s[x4] ] -

                                2              2 2
>    12 EV[s[x3] s[x4]] EV[s[x4] ] + 3 EV[s[x4] ]  +

                         2           2          2
>    4 (2 EV[s[x3] s[x4]]  + EV[s[x3] ] EV[s[x4] ])
             4        2         2         2
case2a= 6 sig  + 2 sig  EV[s[x1] ] - 8 sig  EV[s[x1] s[x3]] +

                      2        2         2            2          2
>    2 EV[s[x1] s[x3]]  + 8 sig  EV[s[x3] ] + EV[s[x1] ] EV[s[x3] ] -

                               2              2 2
>    6 EV[s[x1] s[x3]] EV[s[x3] ] + 3 EV[s[x3] ]  +

          2                                    2
>    4 sig  EV[s[x1] s[x4]] + 2 EV[s[x1] s[x4]]  -
```

```
          2                             2
>    8 sig  EV[s[x3] s[x4]] - 6 EV[s[x3] ] EV[s[x3] s[x4]] +

                      2
>    2 EV[s[x3] s[x4]]  - 2 (2 EV[s[x1] s[x3]] EV[s[x1] s[x4]] +

                2
>       EV[s[x1] ] EV[s[x3] s[x4]]) +

                2
>    4 (EV[s[x3] ] EV[s[x1] s[x4]] + 2 EV[s[x1] s[x3]] EV[s[x3] s[x4]]) +

          2         2            2          2            2          2
>    2 sig  EV[s[x4] ] + EV[s[x1] ] EV[s[x4] ] + EV[s[x3] ] EV[s[x4] ] -

                                                                    2
>    2 (2 EV[s[x1] s[x4]] EV[s[x3] s[x4]] + EV[s[x1] s[x3]] EV[s[x4] ])
             4        2         2         2
case3a= 6 sig  + 2 sig  EV[s[x2] ] + 4 sig  EV[s[x2] s[x3]] +

                      2        2         2            2          2
>    2 EV[s[x2] s[x3]]  + 2 sig  EV[s[x3] ] + EV[s[x2] ] EV[s[x3] ] -

          2                                    2
>    8 sig  EV[s[x2] s[x4]] + 2 EV[s[x2] s[x4]]  -

          2                                    2
>    8 sig  EV[s[x3] s[x4]] + 2 EV[s[x3] s[x4]]  -

                                                    2
>    2 (2 EV[s[x2] s[x3]] EV[s[x2] s[x4]] + EV[s[x2] ] EV[s[x3] s[x4]]) -

                2
>    2 (EV[s[x3] ] EV[s[x2] s[x4]] + 2 EV[s[x2] s[x3]] EV[s[x3] s[x4]]) +

          2         2            2          2            2          2
>    8 sig  EV[s[x4] ] + EV[s[x2] ] EV[s[x4] ] + EV[s[x3] ] EV[s[x4] ] -

                               2                              2
>    6 EV[s[x2] s[x4]] EV[s[x4] ] - 6 EV[s[x3] s[x4]] EV[s[x4] ] +

               2 2
>    3 EV[s[x4] ]  + 4 (2 EV[s[x2] s[x4]] EV[s[x3] s[x4]] +

                                2
```

```
>       EV[s[x2] s[x3]] EV[s[x4] ])
             4        2         2         2
case4a= 4 sig  + 2 sig  EV[s[x1] ] - 4 sig  EV[s[x1] s[x2]] +

          2         2                      2                     2
>    2 sig  EV[s[x2] ] + 2 EV[s[x1] s[x3]]  + 2 EV[s[x2] s[x3]]  +

          2         2            2          2            2          2
>    2 sig  EV[s[x3] ] + EV[s[x1] ] EV[s[x3] ] + EV[s[x2] ] EV[s[x3] ] -

                                                                       2
>    2 (2 EV[s[x1] s[x3]] EV[s[x2] s[x3]] + EV[s[x1] s[x2]] EV[s[x3] ]) +

                       2                     2        2
>    2 EV[s[x1] s[x4]]  + 2 EV[s[x2] s[x4]]  - 4 sig  EV[s[x3] s[x4]] -

                                                      2
>    2 (2 EV[s[x1] s[x3]] EV[s[x1] s[x4]] + EV[s[x1] ] EV[s[x3] s[x4]]) +

>    4 (EV[s[x2] s[x3]] EV[s[x1] s[x4]] +

>       EV[s[x1] s[x3]] EV[s[x2] s[x4]] + EV[s[x1] s[x2]] EV[s[x3] s[x4]])\

>      - 2 (2 EV[s[x2] s[x3]] EV[s[x2] s[x4]] +

                2                           2         2
>       EV[s[x2] ] EV[s[x3] s[x4]]) + 2 sig  EV[s[x4] ] +

             2          2            2          2
>    EV[s[x1] ] EV[s[x4] ] + EV[s[x2] ] EV[s[x4] ] -

                                                                       2
>    2 (2 EV[s[x1] s[x4]] EV[s[x2] s[x4]] + EV[s[x1] s[x2]] EV[s[x4] ])
              4         2              2         2
case1b= 12 sig  + 24 sig  G[0] + 8 G[0]  - 24 sig  G[-x3 + x4] -

                                        2          2                2
>    24 G[0] G[-x3 + x4] + 4 G[-x3 + x4]  + 4 (G[0]  + 2 G[-x3 + x4] )
             4         2              2        2
case2b= 6 sig  + 12 sig  G[0] + 6 G[0]  - 8 sig  G[-x1 + x3] -

                                       2        2
>    6 G[0] G[-x1 + x3] + 2 G[-x1 + x3]  + 4 sig  G[-x1 + x4] +

                   2        2
```

```
>    2 G[-x1 + x4]  - 8 sig  G[-x3 + x4] - 6 G[0] G[-x3 + x4] +

                   2
>    2 G[-x3 + x4]  - 2 (2 G[-x1 + x3] G[-x1 + x4] + G[0] G[-x3 + x4]) +

>    4 (G[0] G[-x1 + x4] + 2 G[-x1 + x3] G[-x3 + x4]) -

>    2 (G[0] G[-x1 + x3] + 2 G[-x1 + x4] G[-x3 + x4])
              4         2              2        2
case3b= 6 sig  + 12 sig  G[0] + 6 G[0]  + 4 sig  G[-x2 + x3] +

                   2        2
>    2 G[-x2 + x3]  - 8 sig  G[-x2 + x4] - 6 G[0] G[-x2 + x4] +

                   2        2
>    2 G[-x2 + x4]  - 8 sig  G[-x3 + x4] - 6 G[0] G[-x3 + x4] +

                   2
>    2 G[-x3 + x4]  - 2 (2 G[-x2 + x3] G[-x2 + x4] + G[0] G[-x3 + x4]) -

>    2 (G[0] G[-x2 + x4] + 2 G[-x2 + x3] G[-x3 + x4]) +

>    4 (G[0] G[-x2 + x3] + 2 G[-x2 + x4] G[-x3 + x4])
              4        2              2        2
case4b= 4 sig  + 8 sig  G[0] + 4 G[0]  - 4 sig  G[-x1 + x2] +

                   2                2
>    2 G[-x1 + x3]  + 2 G[-x2 + x3]  -

                                                                   2
>    2 (G[0] G[-x1 + x2] + 2 G[-x1 + x3] G[-x2 + x3]) + 2 G[-x1 + x4]  +

                   2
>    2 G[-x2 + x4]  - 2 (G[0] G[-x1 + x2] + 2 G[-x1 + x4] G[-x2 + x4]) -

           2
>    4 sig  G[-x3 + x4] - 2 (2 G[-x1 + x3] G[-x1 + x4] +

>       G[0] G[-x3 + x4]) - 2 (2 G[-x2 + x3] G[-x2 + x4] +

>       G[0] G[-x3 + x4]) + 4 (G[-x2 + x3] G[-x1 + x4] +

>       G[-x1 + x3] G[-x2 + x4] + G[-x1 + x2] G[-x3 + x4])
               4         2               2         2
case1c= 12 sig  + 24 sig  G[0] + 12 G[0]  - 24 sig  G[p] - 24 G[0] G[p] +
```

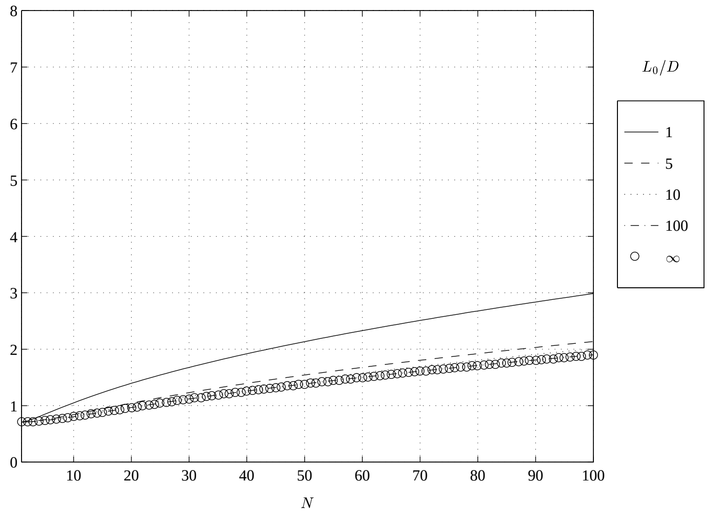
```
               2
>    12 G[p]
                4          2                2          2
case2c= 6 sig  + 12 sig  G[0] + 6 G[0]  - 16 sig  G[p] - 16 G[0] G[p] +

               2        2                                                      2
>    12 G[p]  + 4 sig  G[2 p] + 4 G[0] G[2 p] - 8 G[p] G[2 p] + 2 G[2 p]
                4          2                2          2
case3c= 6 sig  + 12 sig  G[0] + 6 G[0]  - 16 sig  G[p] - 16 G[0] G[p] +

               2        2                                                      2
>    12 G[p]  + 4 sig  G[2 p] + 4 G[0] G[2 p] - 8 G[p] G[2 p] + 2 G[2 p]
                4         2               2         2
case4c= 4 sig  + 8 sig  G[0] + 4 G[0]  - 8 sig  G[p] - 8 G[0] G[p] +

              2                  2
>    4 G[p]  + 8 G[-x1 + x3]  - 8 G[-x1 + x3] G[-x2 + x3] +

                     2
>    2 G[-x2 + x3]  - 8 G[-x1 + x3] G[-x1 + x4] +

                                                     2
>    4 G[-x2 + x3] G[-x1 + x4] + 2 G[-x1 + x4]
```

N

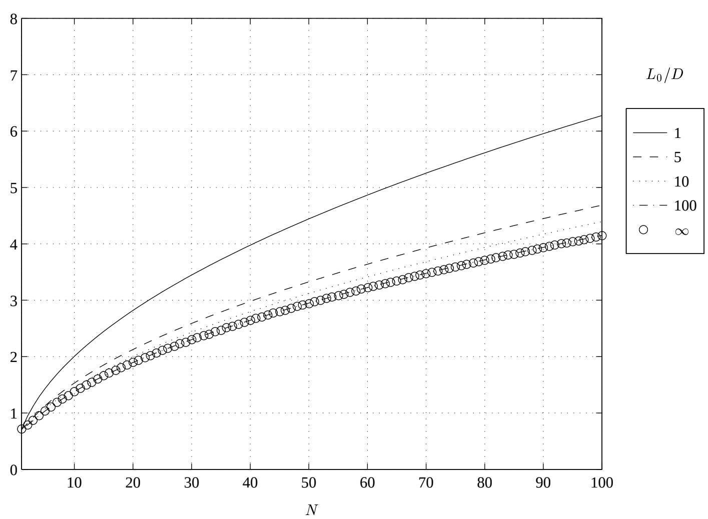
α /

		$\vec{\rho}$			
		$D, \circ$	$L_0/D \quad \{ \quad \infty\}$	$\sigma_n^2/ \ _s \quad \{ \quad \}$ $\lvert\vec{v}\tau/D\rvert \quad \{ \quad \}$	$\theta_0 \quad \circ$ $\alpha \quad /$
		$D, \circ$	$L_0/D \quad \{ \quad \infty\}$	$\sigma_n^2/ \ _s \quad \{ \quad \}$ $\lvert\vec{v}\tau/D\rvert \quad \{ \quad \}$	$\theta_0 \quad \circ$ $\alpha \quad /$
		$D, \circ$	$\sigma_n^2/ \ _s \quad \{ \quad \}$	$L_0/D \quad \{ \quad \infty\}$ $\lvert\vec{v}\tau/D\rvert \quad \{ \quad \}$	$\theta_0 \quad \circ$ $\alpha \quad /$
		$D, \circ$	$\lvert\vec{v}\tau/D\rvert \quad \{ \quad \}$	$L_0/D \quad \{ \quad \infty\}$ $\sigma_n^2/ \ _s \quad \{ \quad \}$	$\theta_0 \quad \circ$ $\alpha \quad /$
		D, θ_0	$\theta_0 \quad \{ \circ \quad \circ \quad \circ\}$	$\lvert\vec{v}\tau/D\rvert \quad \{ \quad \}$ $\sigma_n^2/ \ _s \quad \{ \quad \}$	$L_0/D \quad \infty$ $\alpha \quad /$

N



N
$\vec{\rho}$. D, . $^\circ$ α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



N
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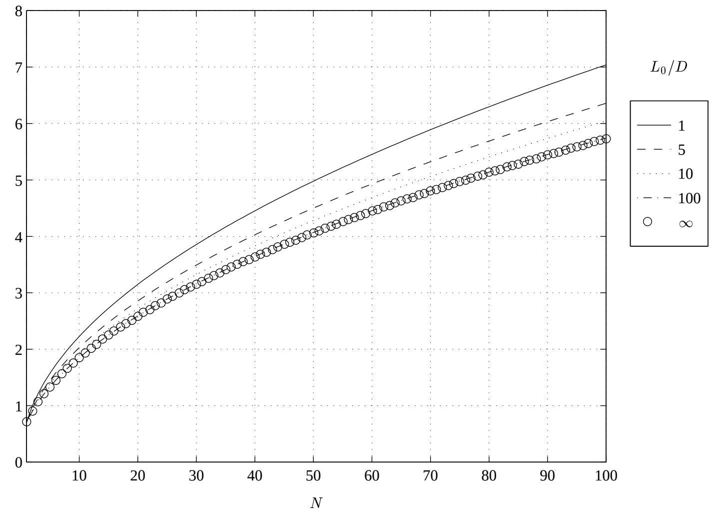
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



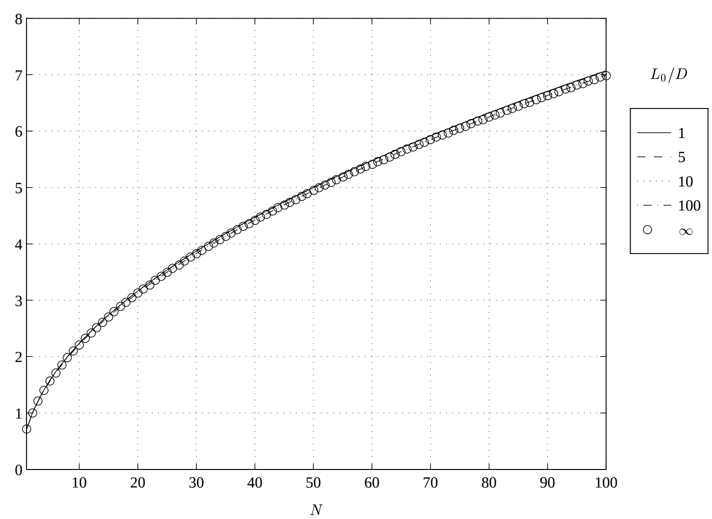
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



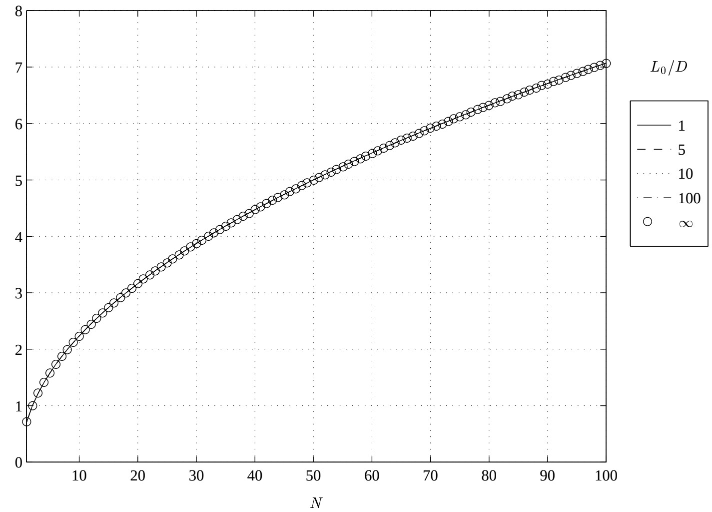
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



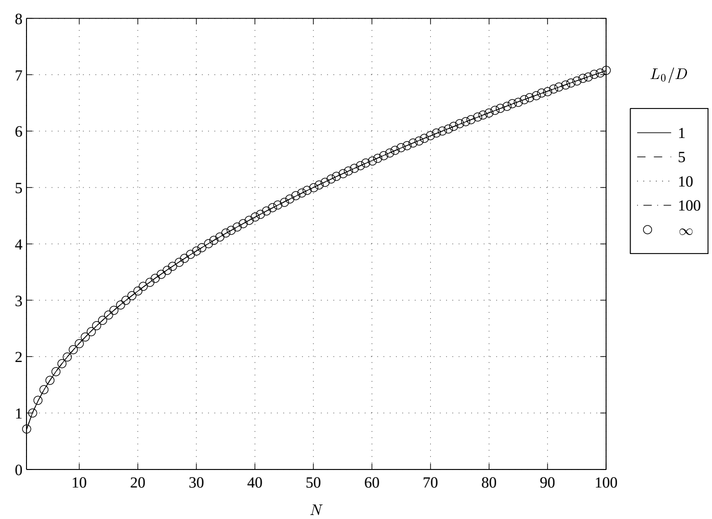
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ $_s$

L_0/D ∞



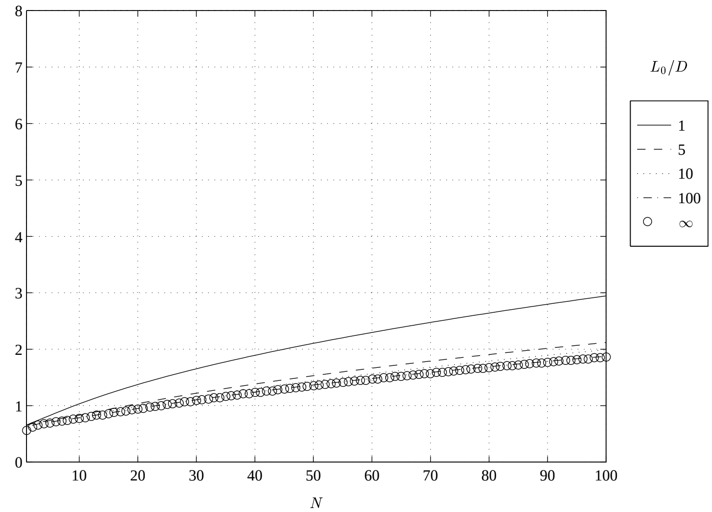
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ $_s$

L_0/D ∞



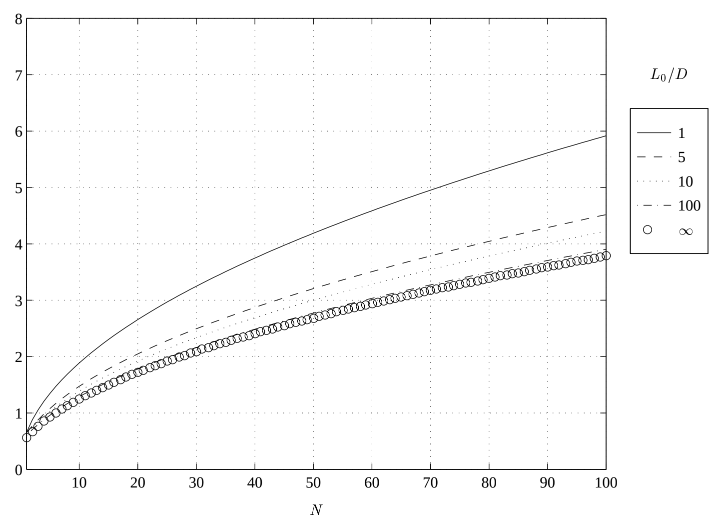
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



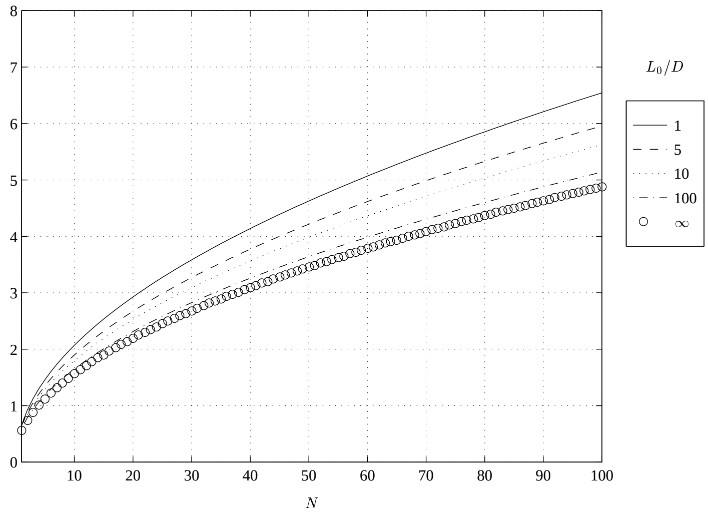
N

$\vec{\rho}$. D, . $^\circ$ α .

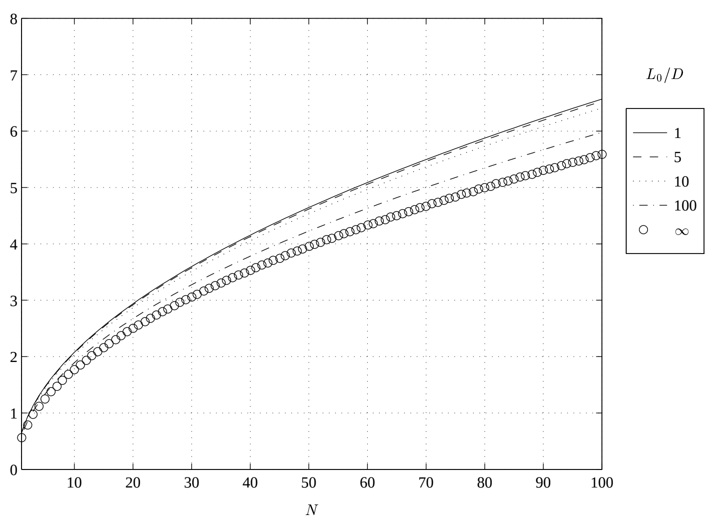
$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

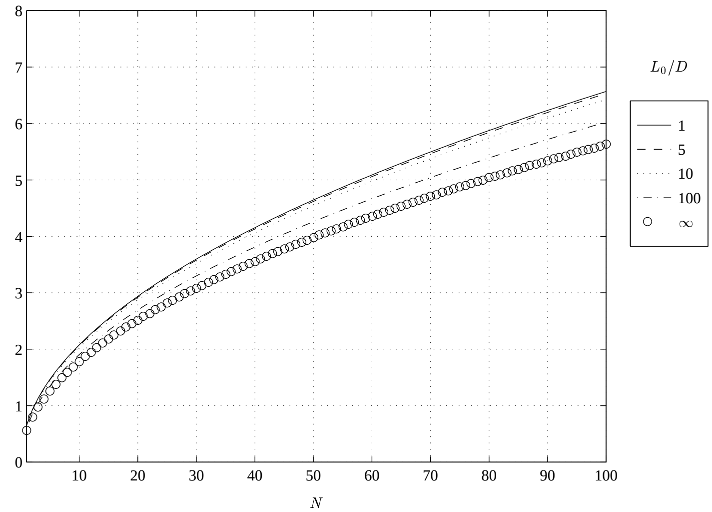
L_0/D ∞



N
$\vec{\rho}$. D, . $^\circ$ α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s
L_0/D ∞



N
$\vec{\rho}$. D, . $^\circ$ α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/$ s
L_0/D ∞



N
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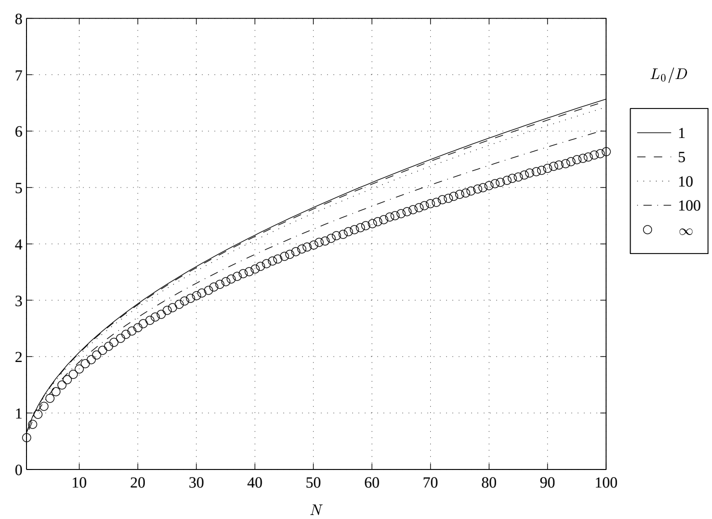
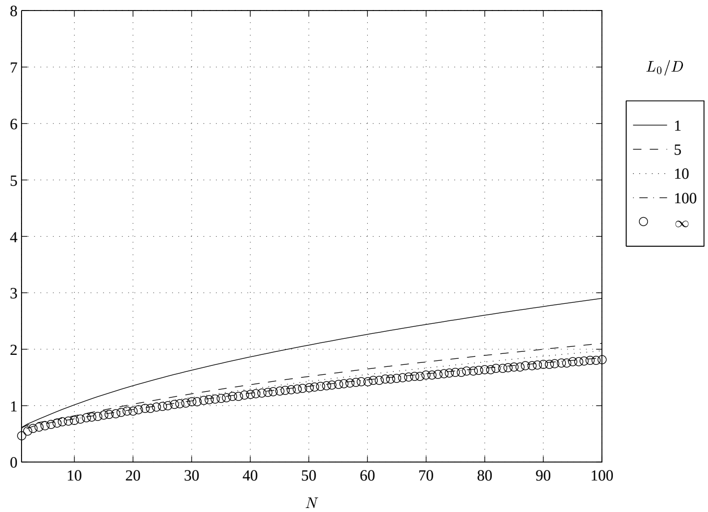
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



N
$\vec{\rho}$. D, . $^\circ$ α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



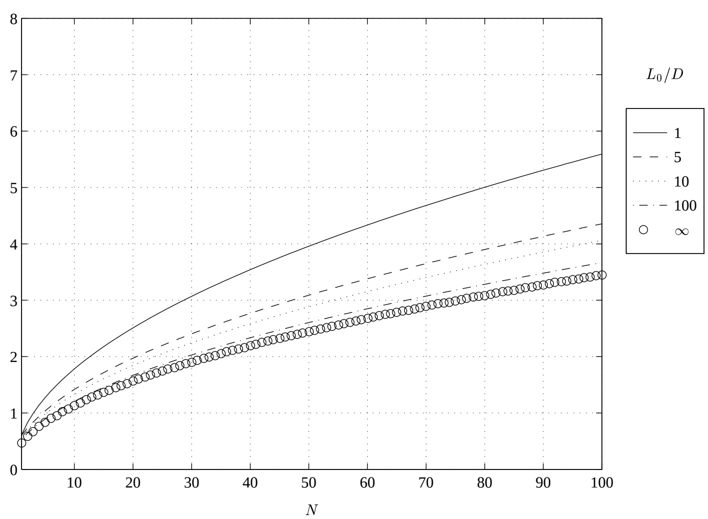
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/\ _s$

L_0/D ∞



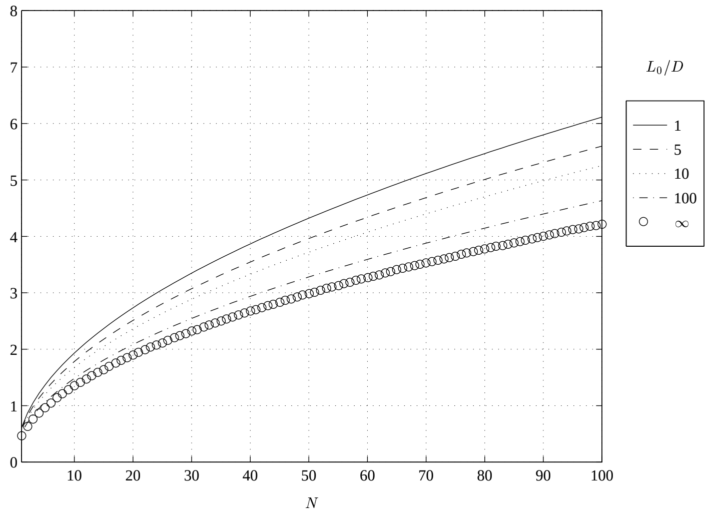
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/\ _s$

L_0/D ∞



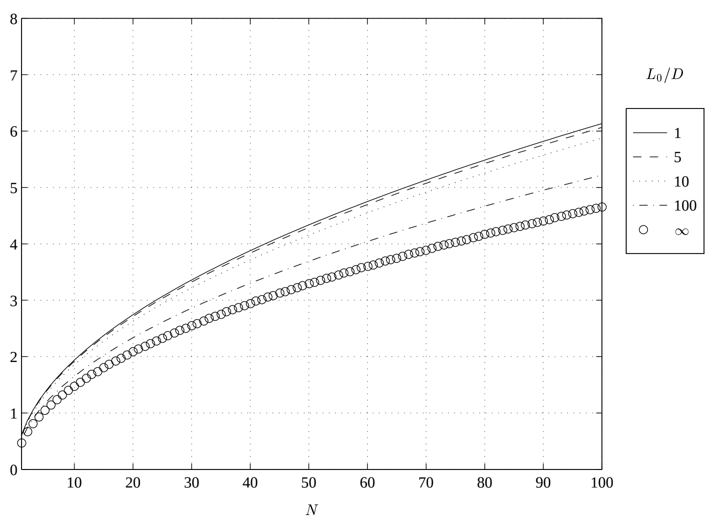
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



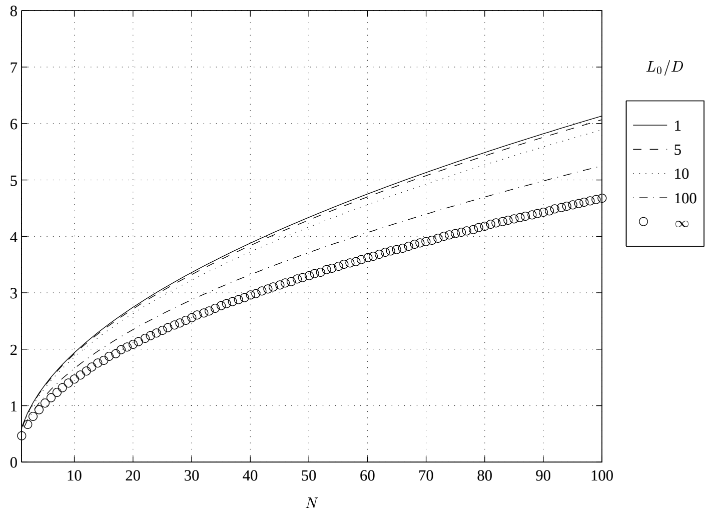
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



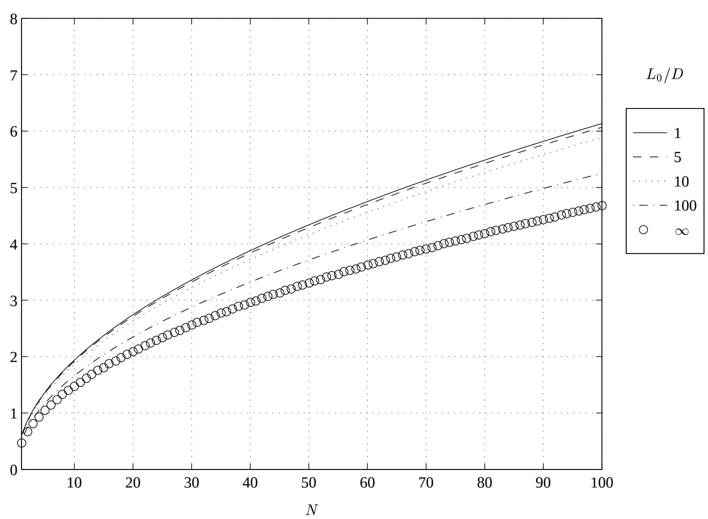
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



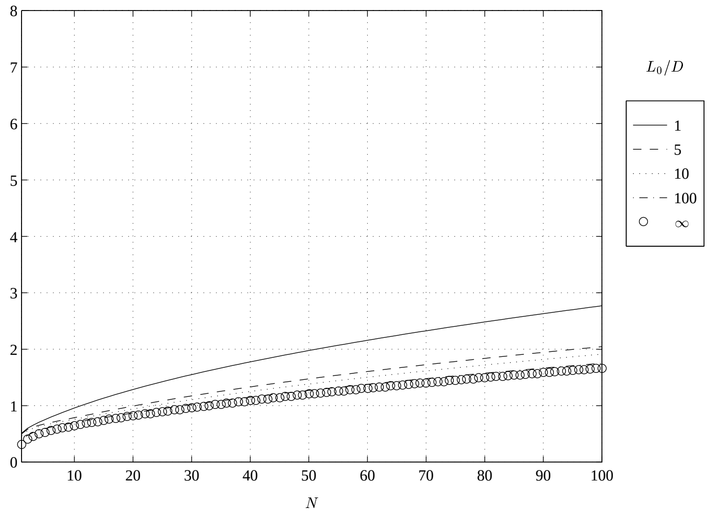
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



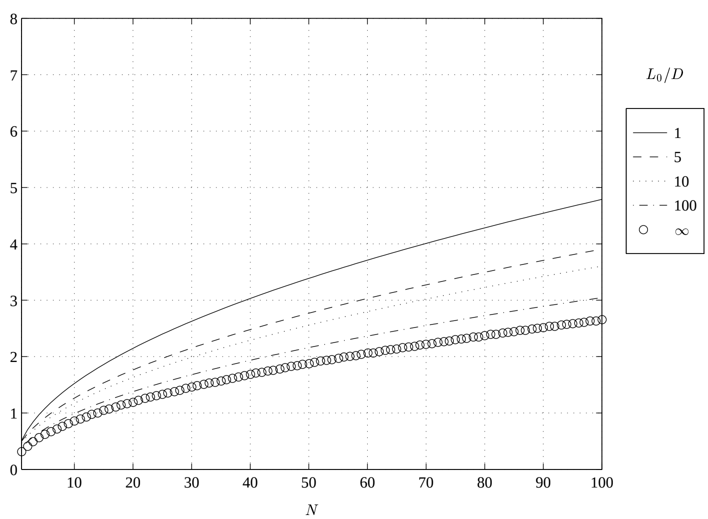
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/\ _s$

L_0/D ∞



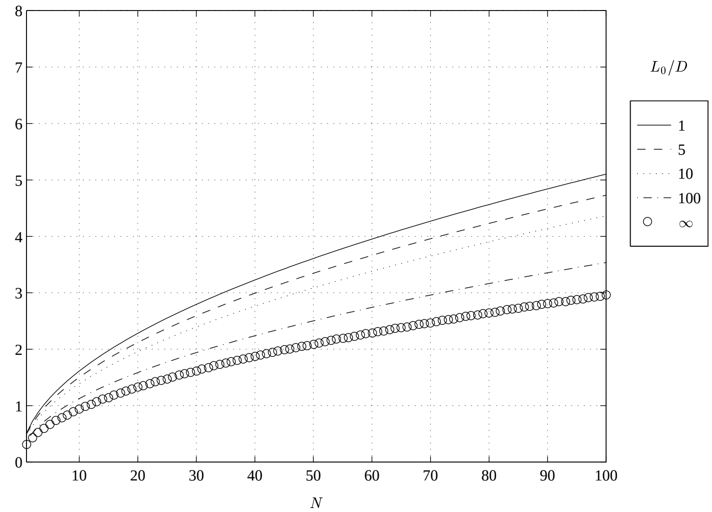
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/\ _s$

L_0/D ∞



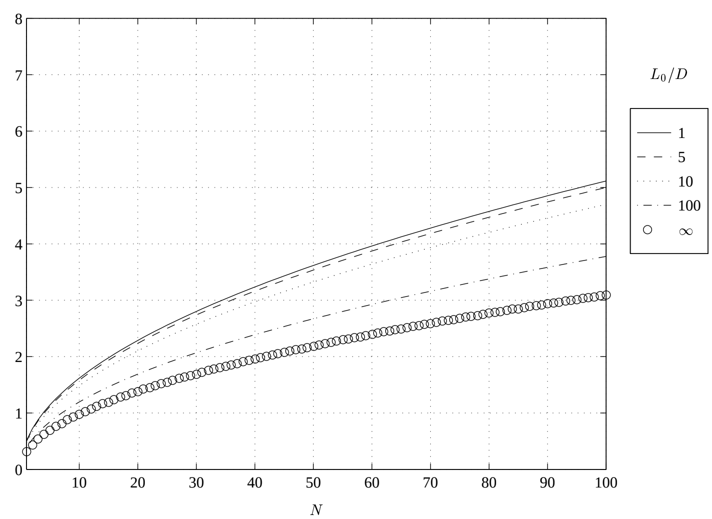
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



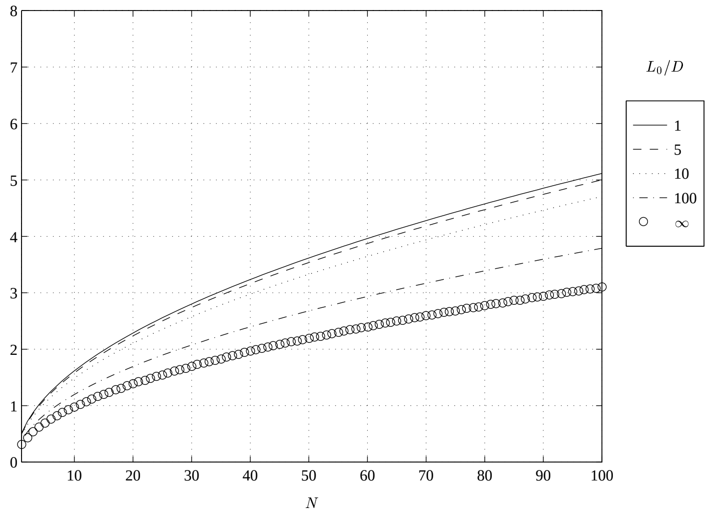
N

$\vec{\rho}$. D, . $^\circ$ α .

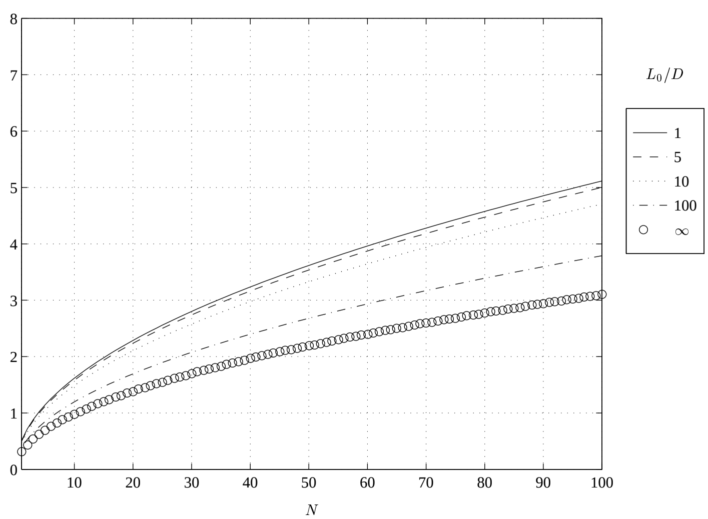
$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

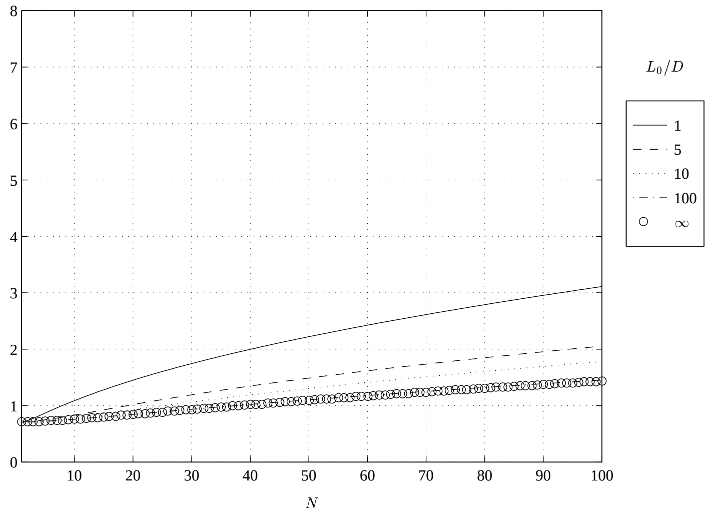
L_0/D ∞



N
$\vec{\rho}$. D, . ° α .
$\vec{v}\tau$. D, . °
$\sigma_n^2/$ s
L_0/D ∞



N
$\vec{\rho}$. D, . ° α .
$\vec{v}\tau$. D, . °
$\sigma_n^2/$ s
L_0/D ∞



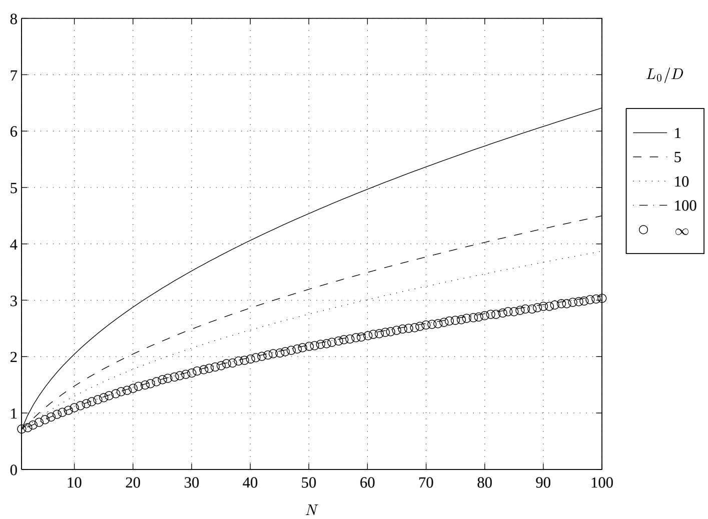
N

$\vec{\rho}$. D, . $^\circ$ α .

$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



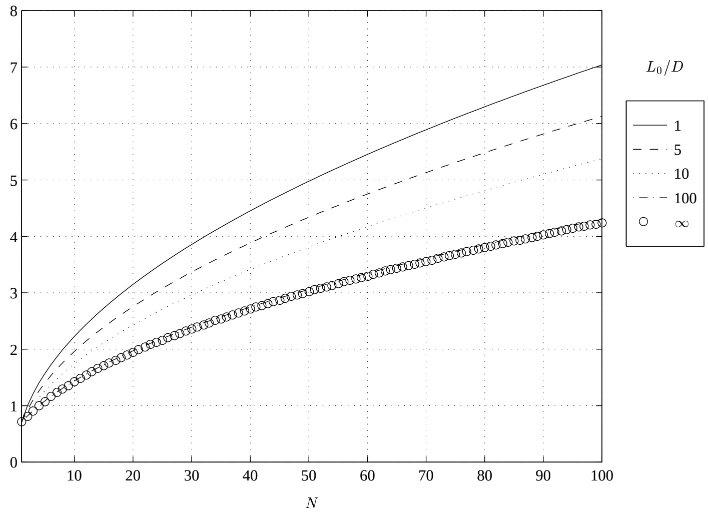
N

$\vec{\rho}$. D, . $^\circ$ α .

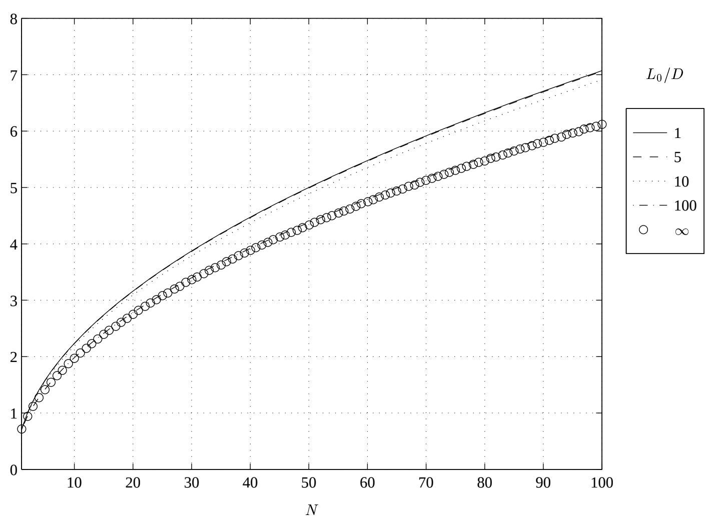
$\vec{v}\tau$. D, . $^\circ$

$\sigma_n^2/$ s

L_0/D ∞



N
$\vec{\rho}$. D, . $^\circ$ α .
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



N
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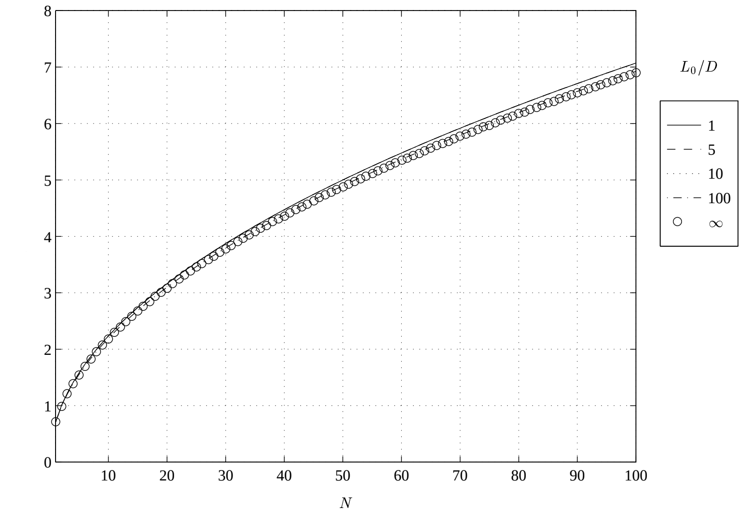
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



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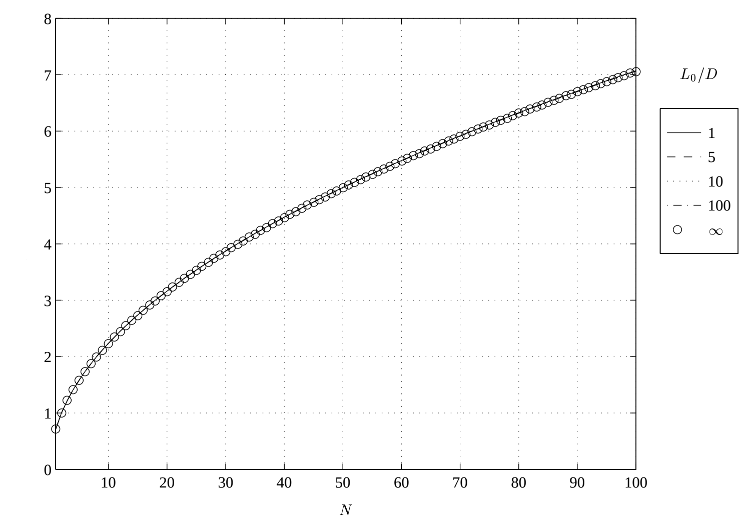
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ \ _s$
L_0/D ∞



N
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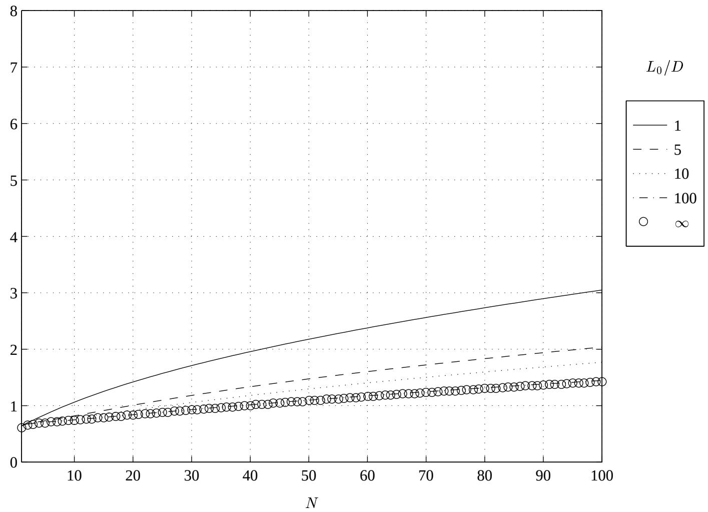
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ \ _s$
L_0/D ∞



N

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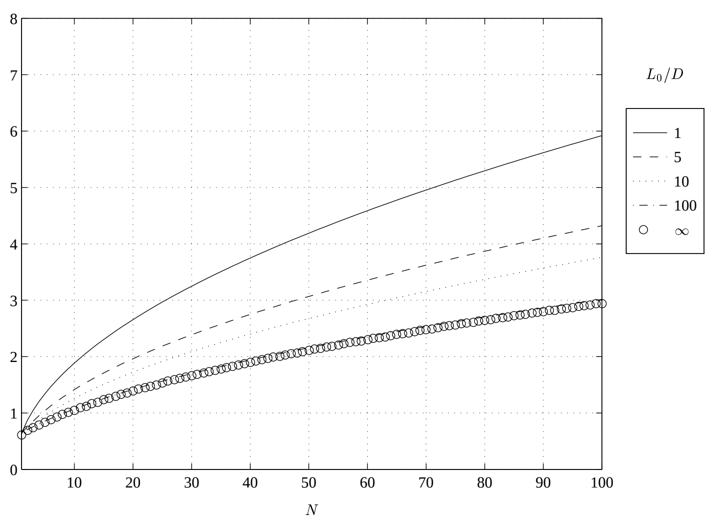
$\vec{v}\tau$. D, . $^\circ$
$\sigma_n^2/\ _s$
L_0/D ∞



N

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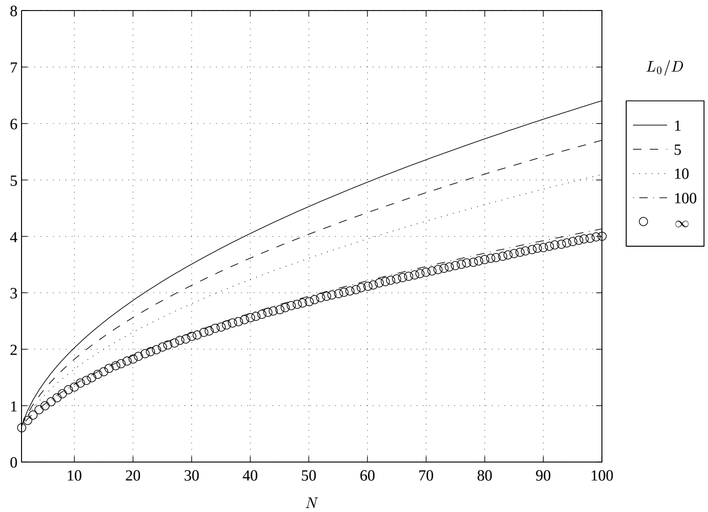
$\sigma_n^2/\ _s$
L_0/D ∞



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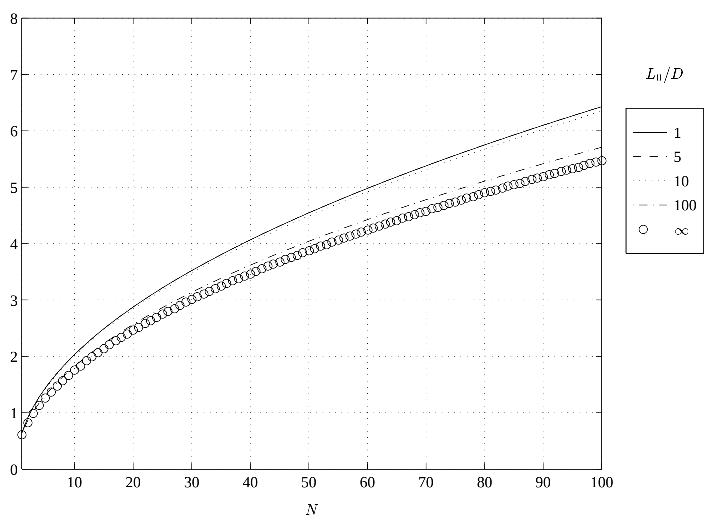

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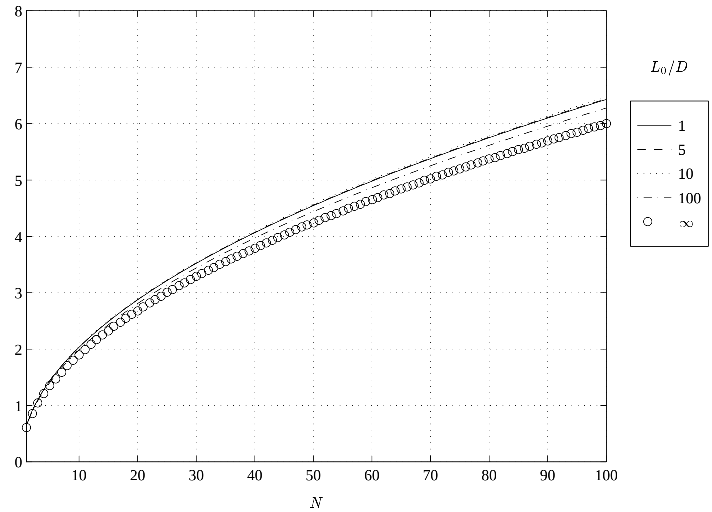
L_0/D ∞



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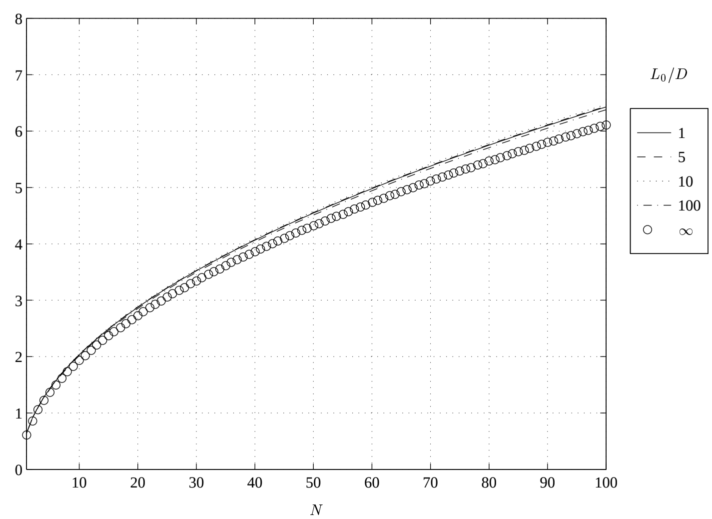

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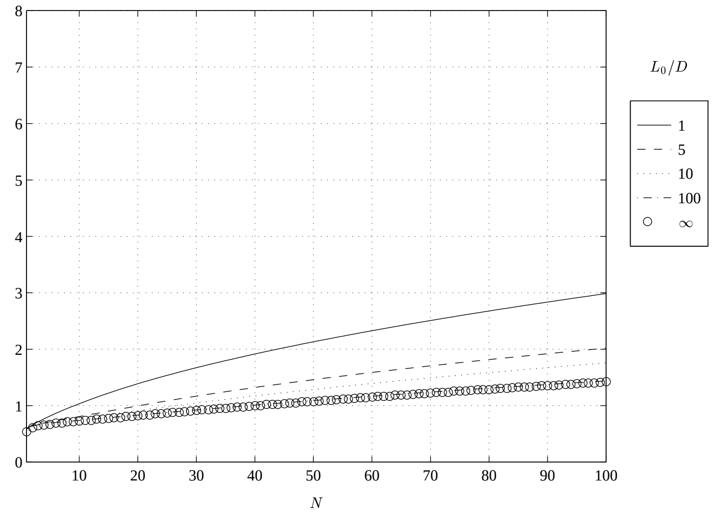

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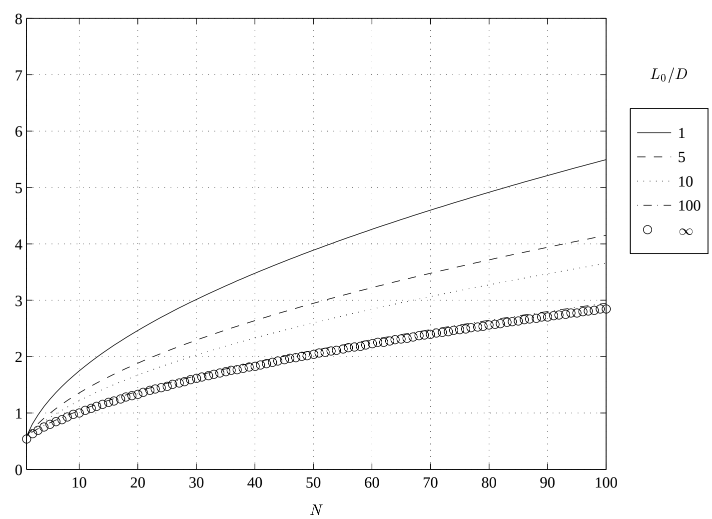
L_0/D ∞



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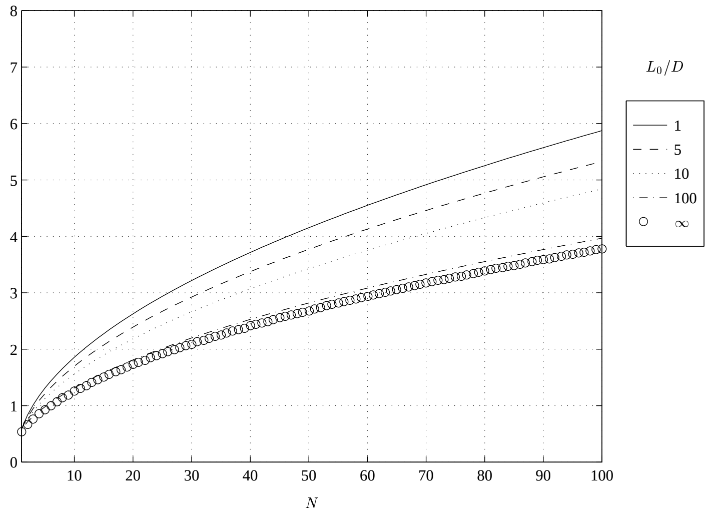

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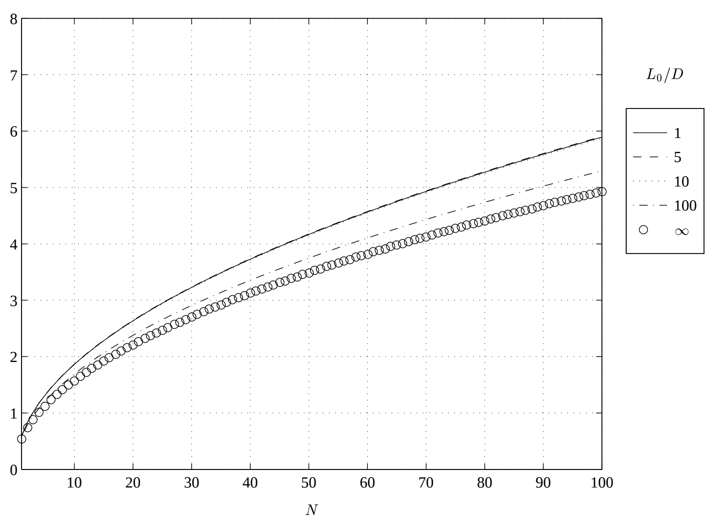
L_0/D ∞



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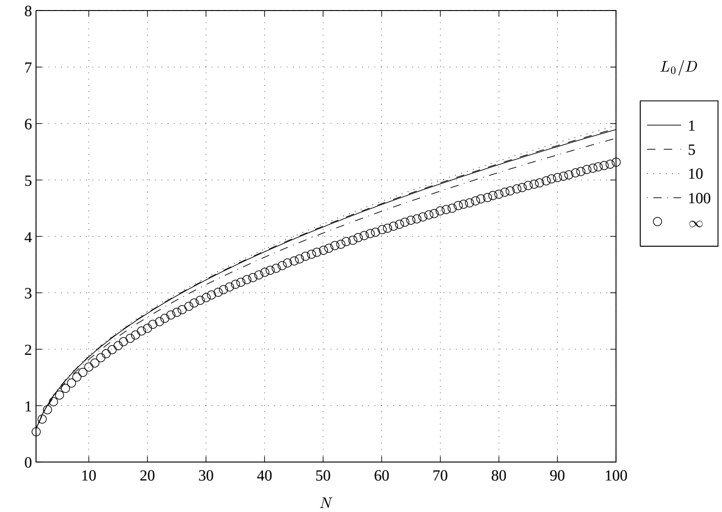

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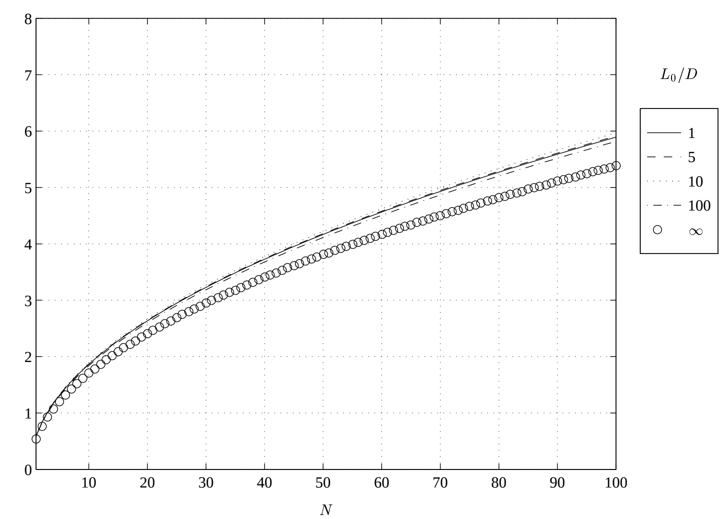

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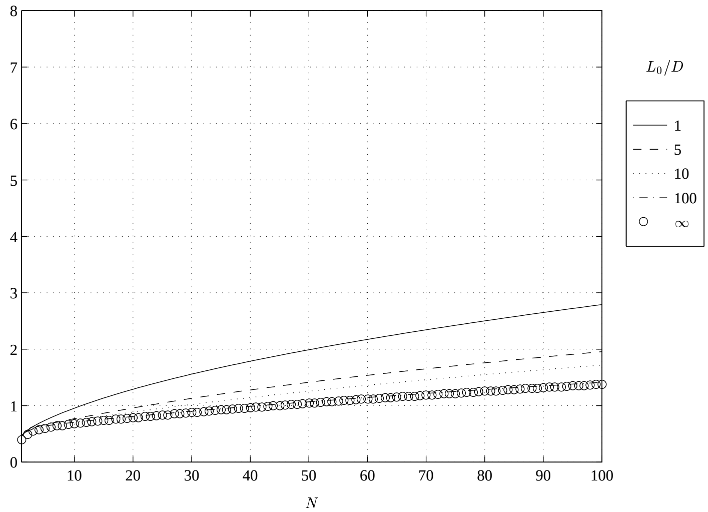

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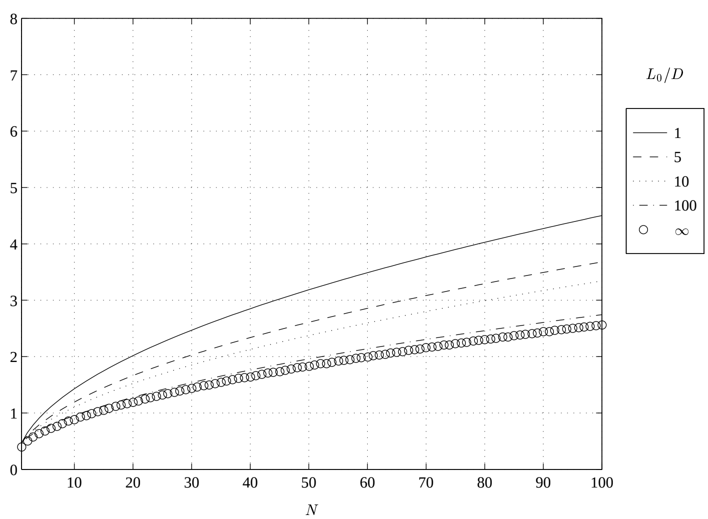
L_0/D ∞



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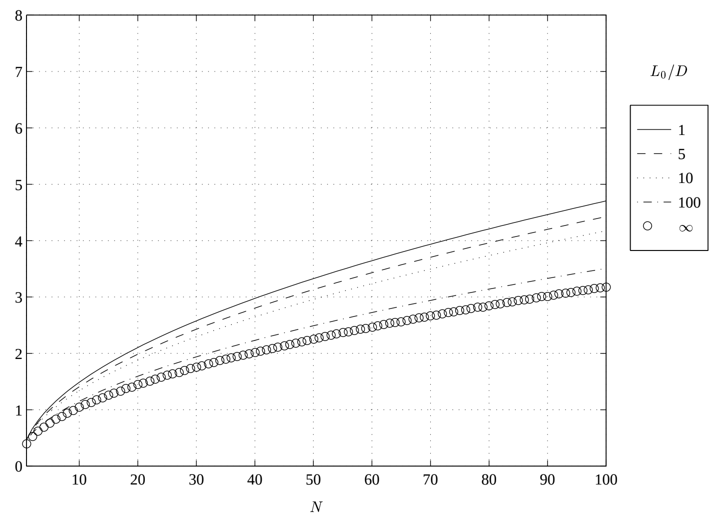
L_0/D ∞



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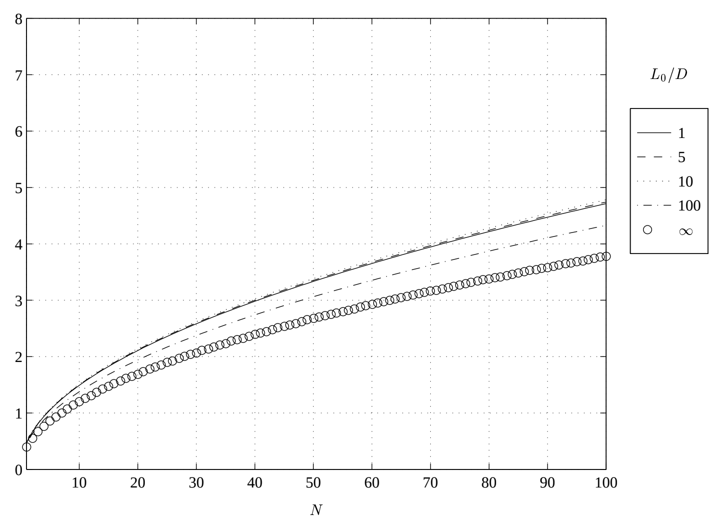
$\sigma_n^2/\ _s$
L_0/D ∞



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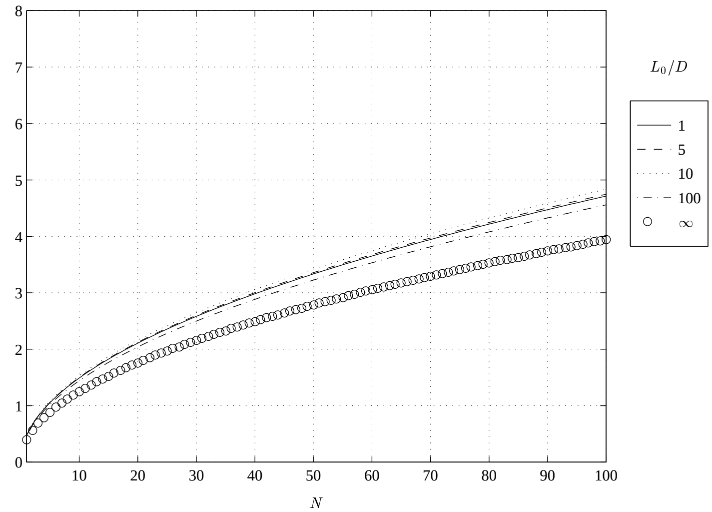

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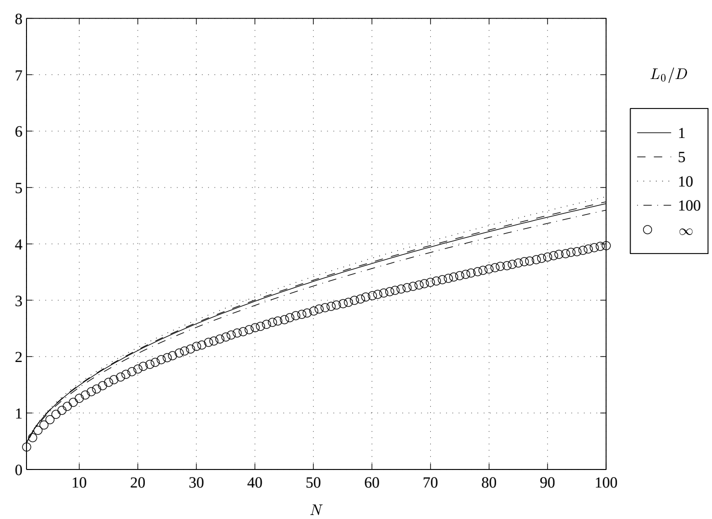
L_0/D ∞



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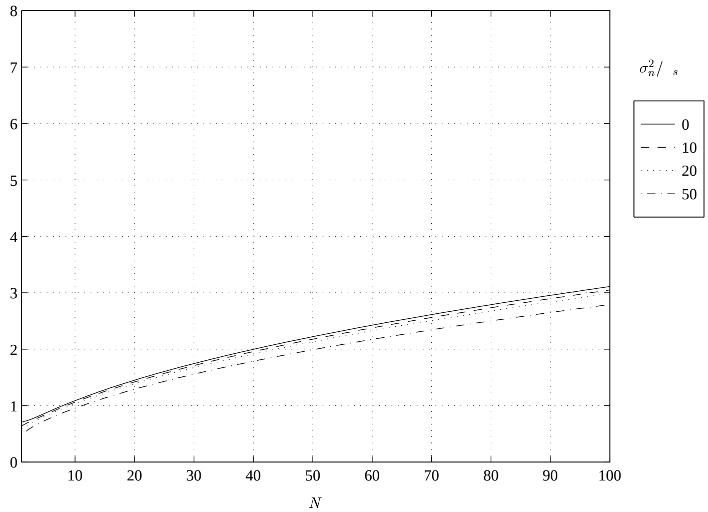
$\sigma_n^2/$ s

L_0/D ∞



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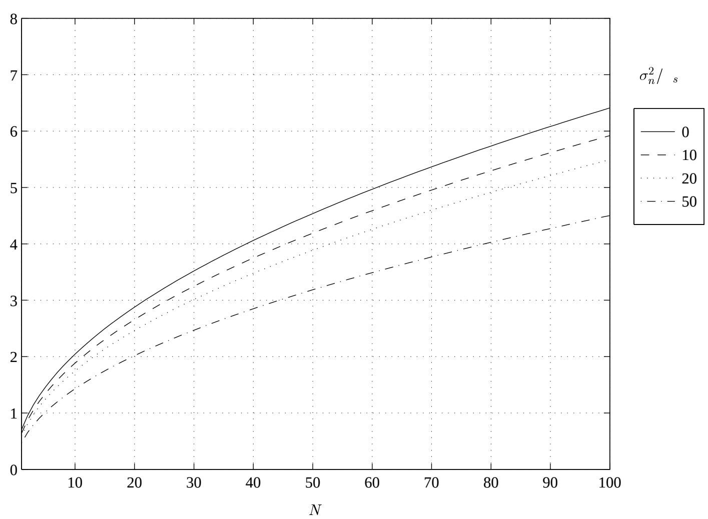
$\sigma_n^2/\ _s$



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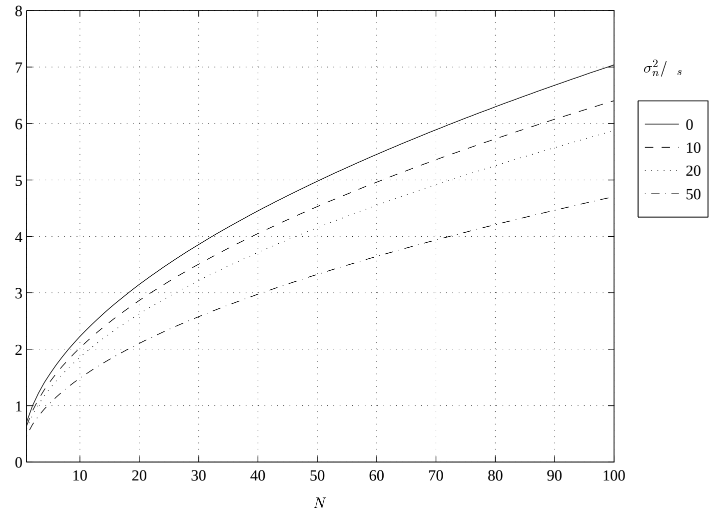

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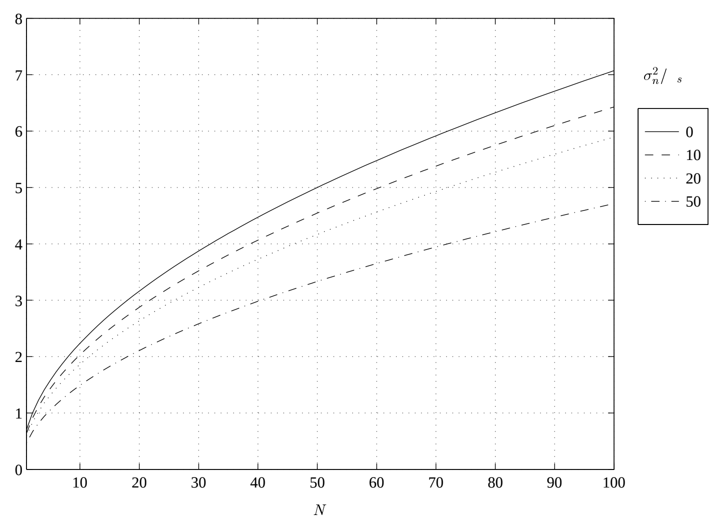

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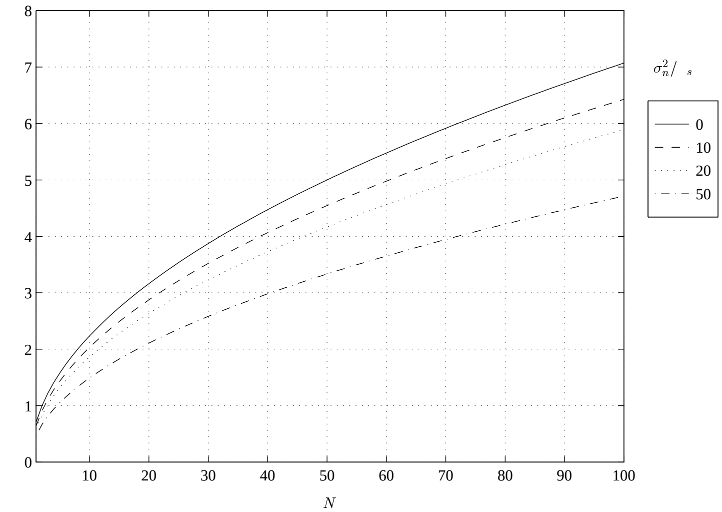

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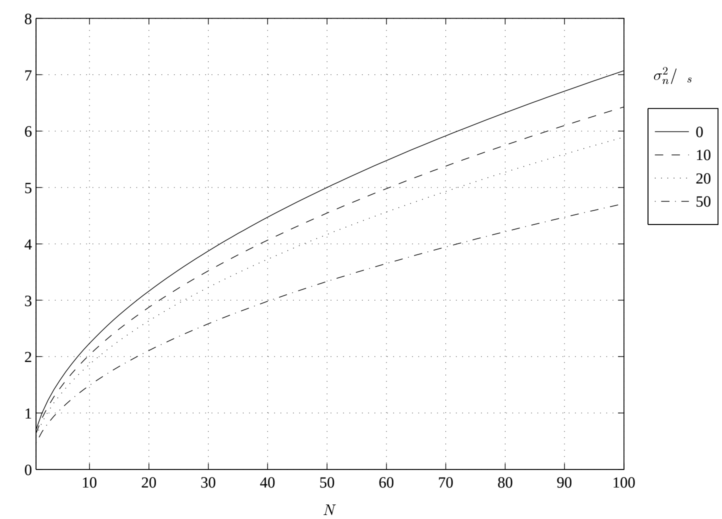

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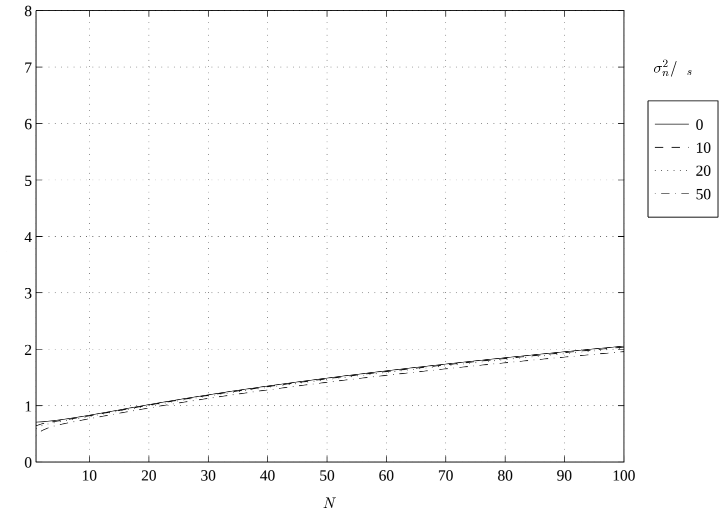

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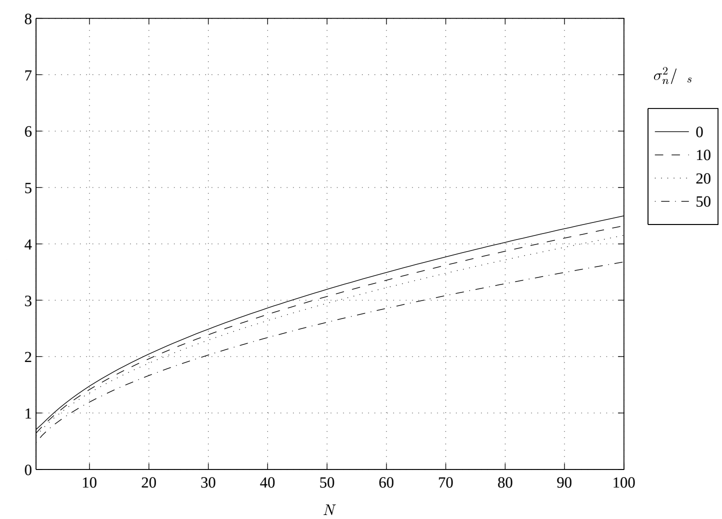

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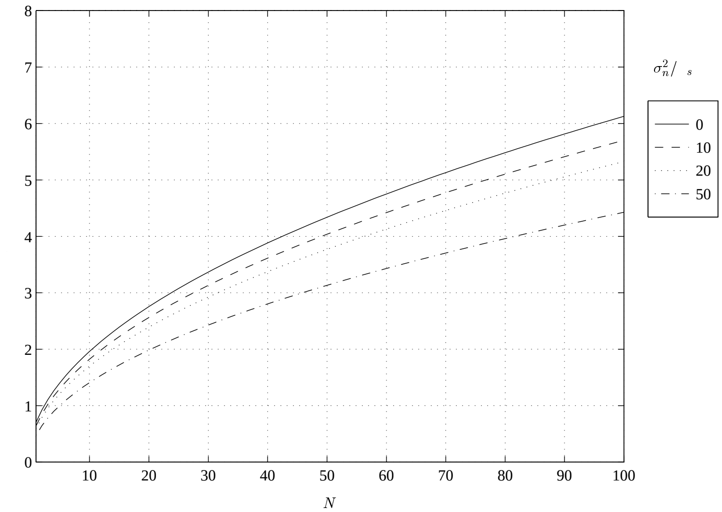

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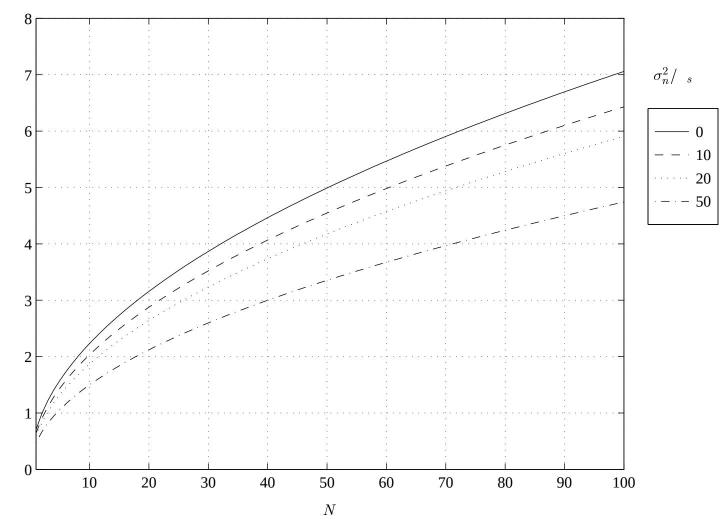

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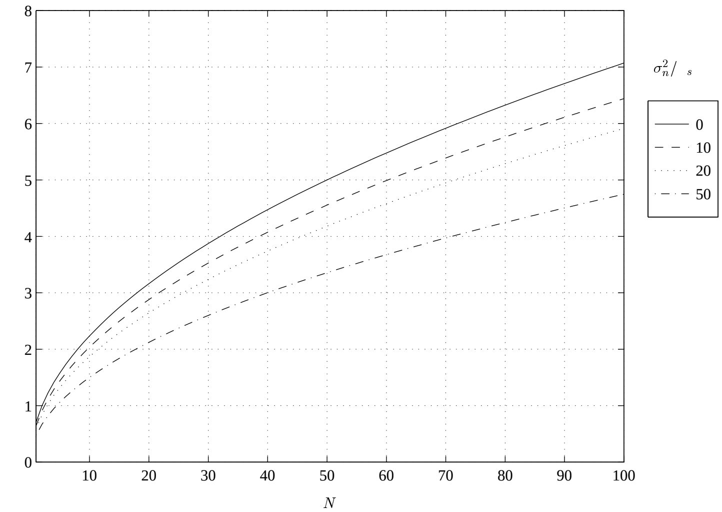

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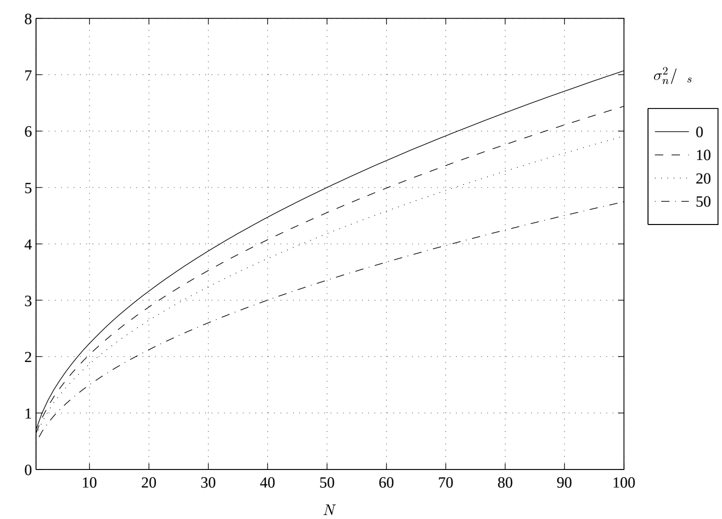

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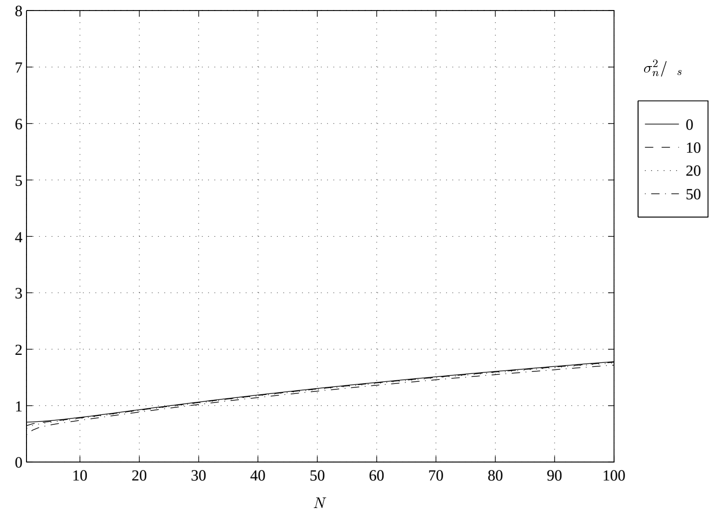

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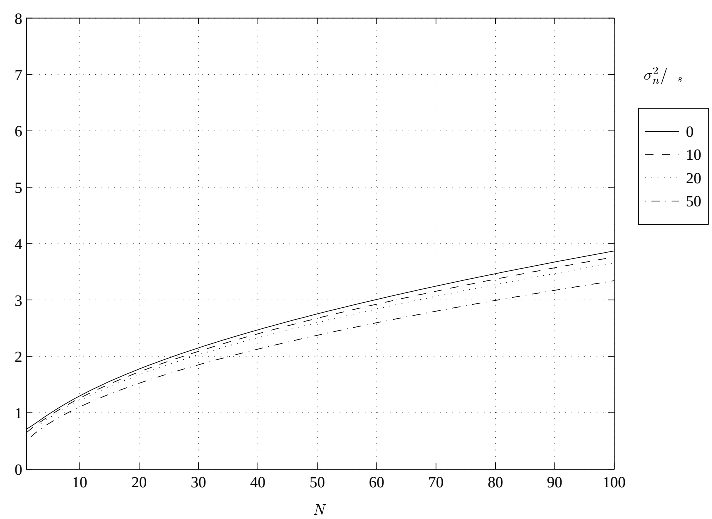

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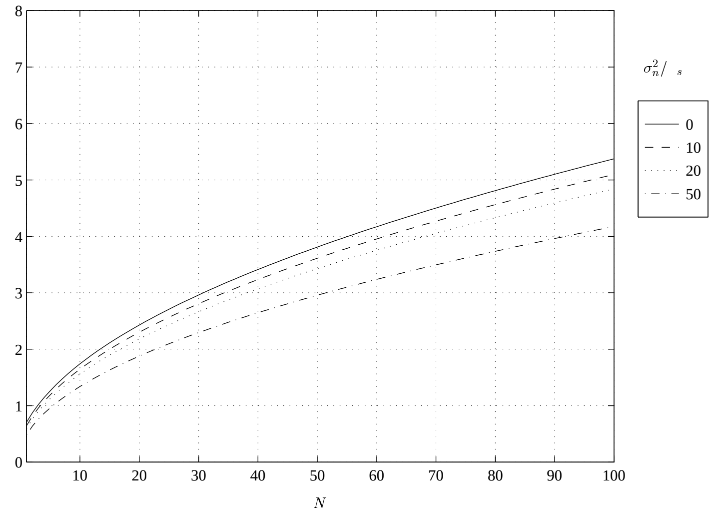

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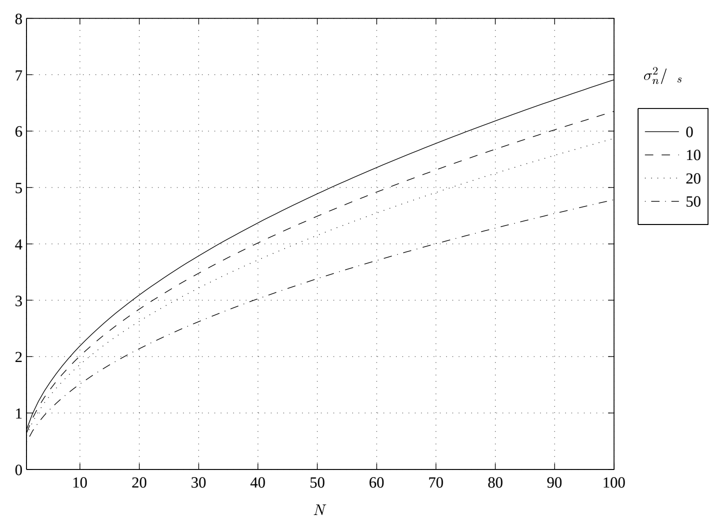

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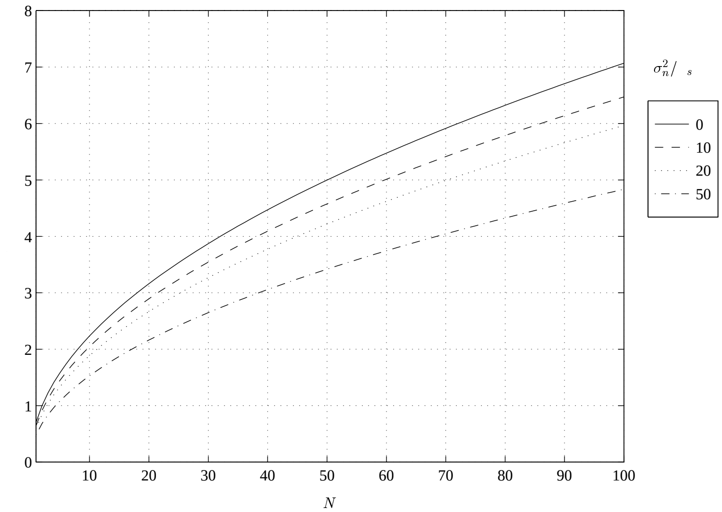

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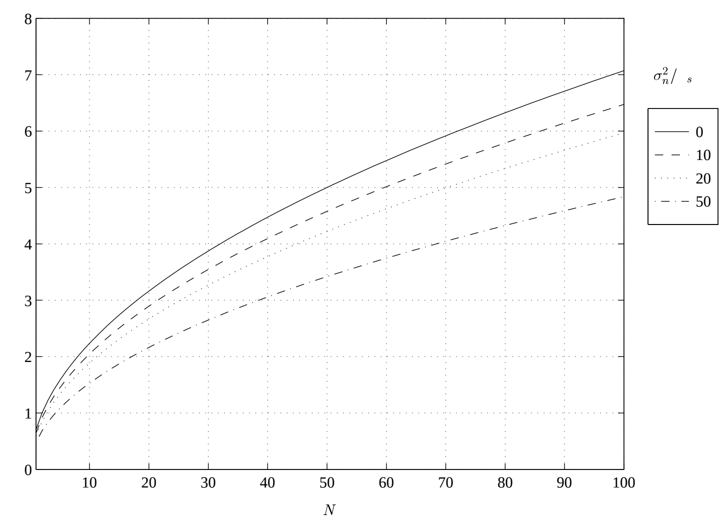

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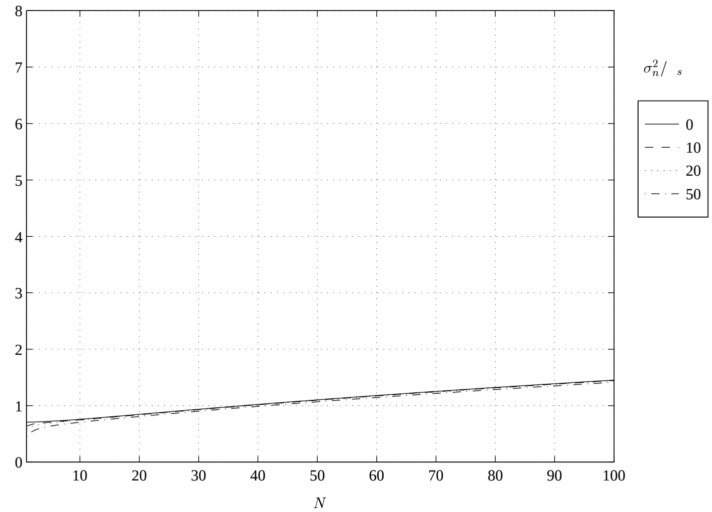

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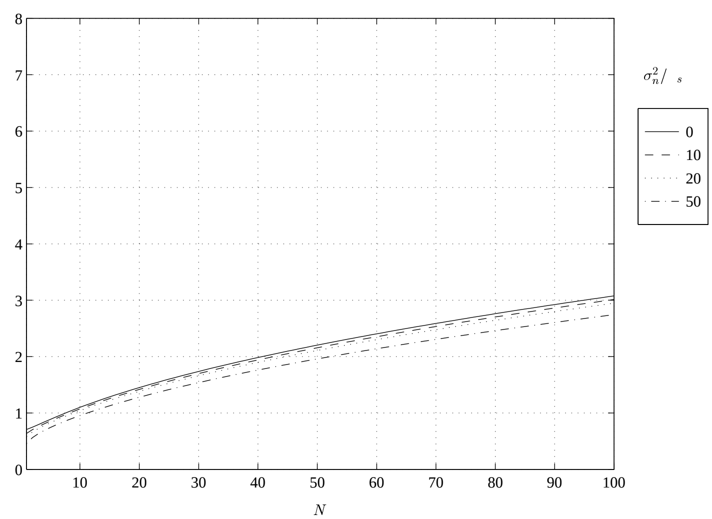

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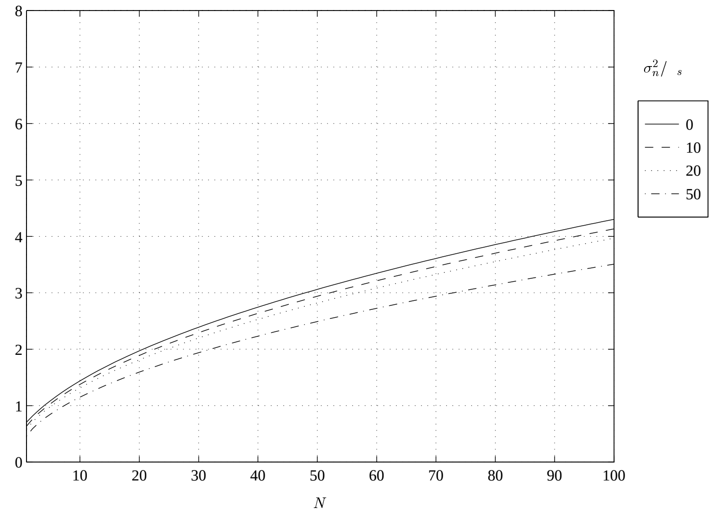

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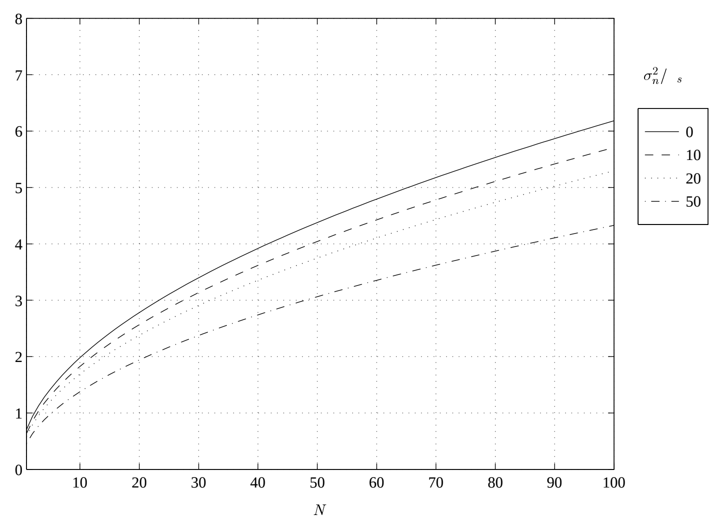

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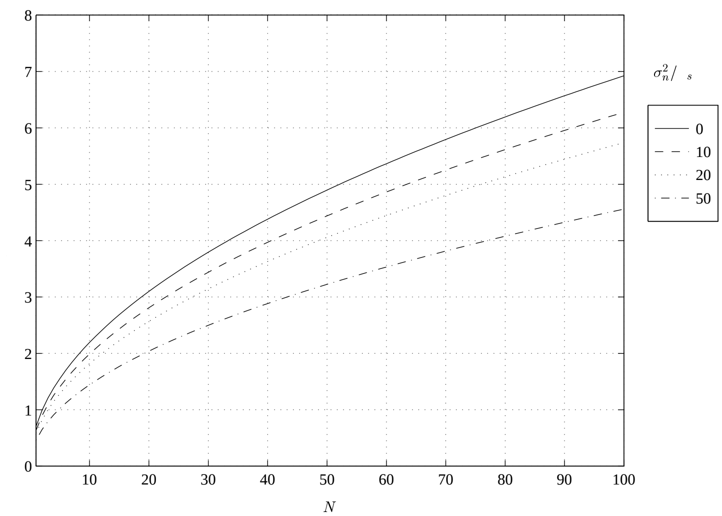

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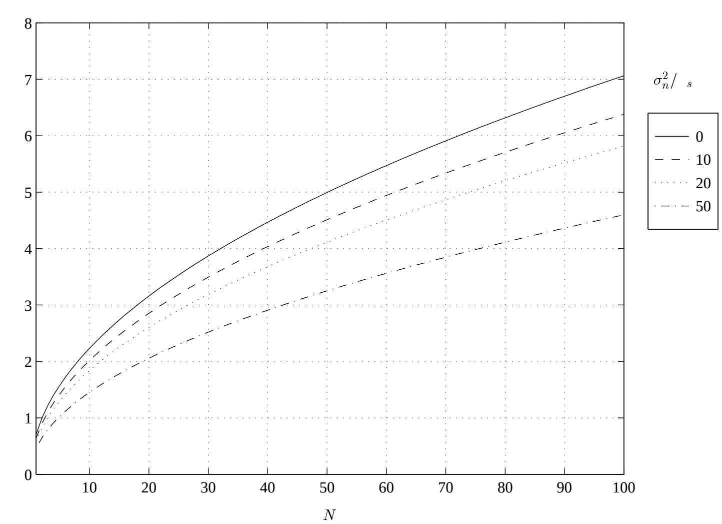

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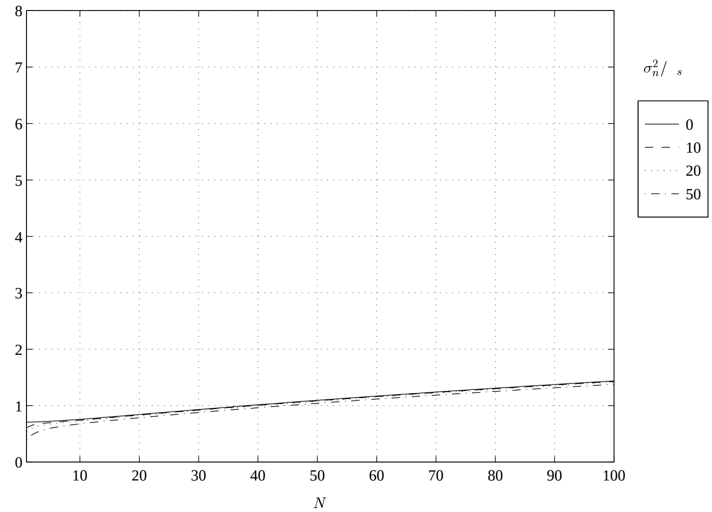

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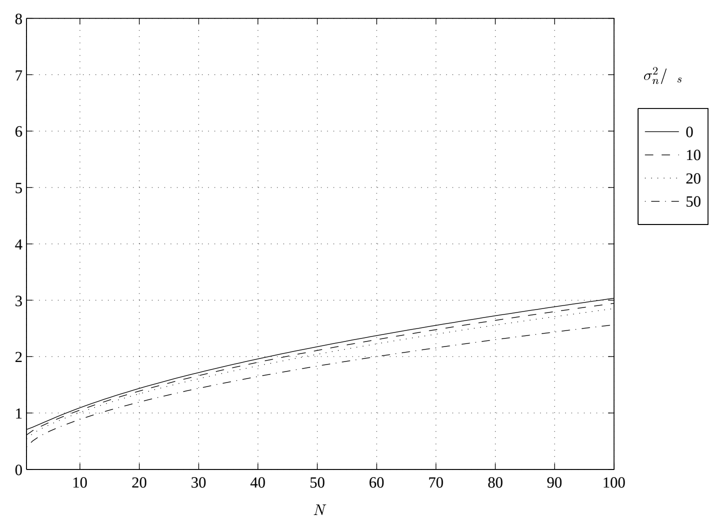

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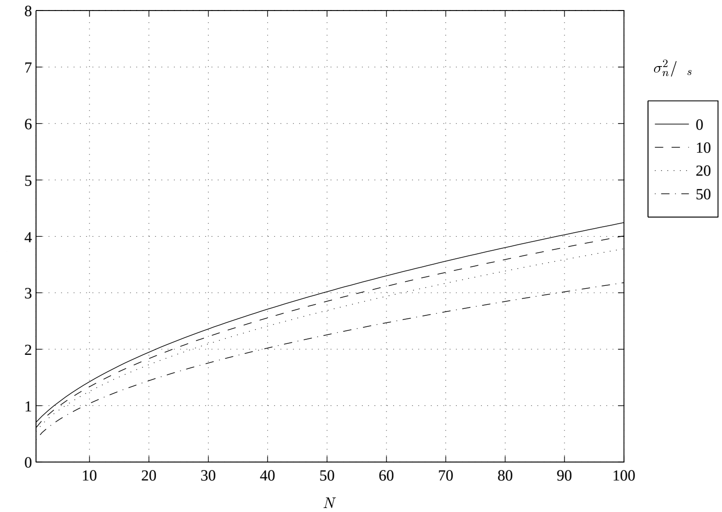

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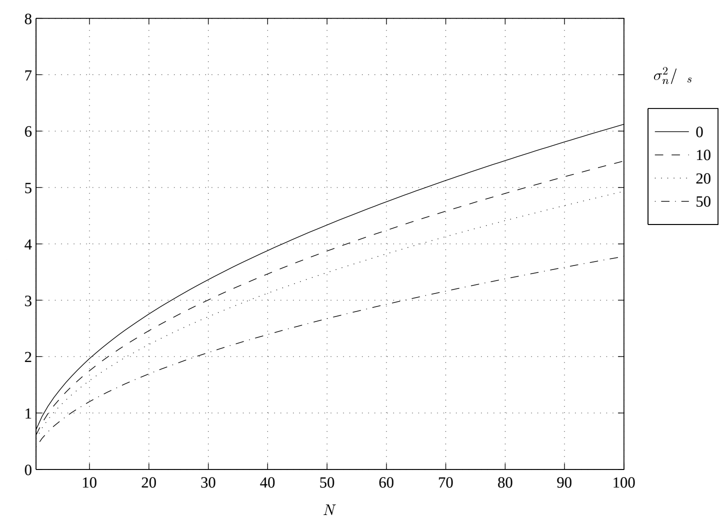

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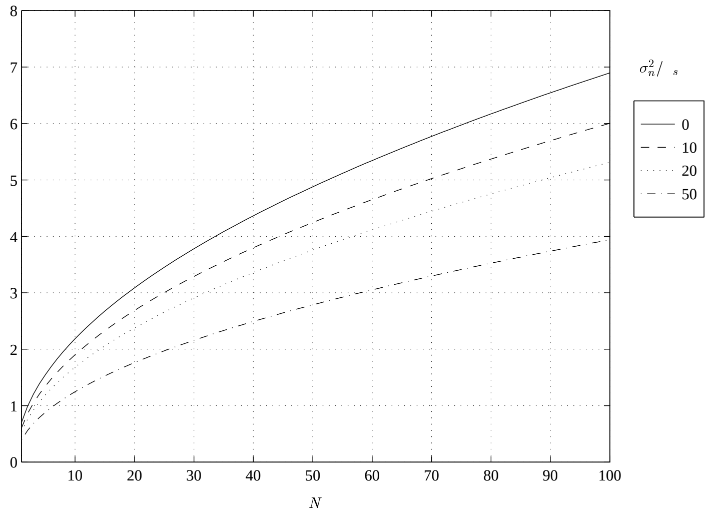

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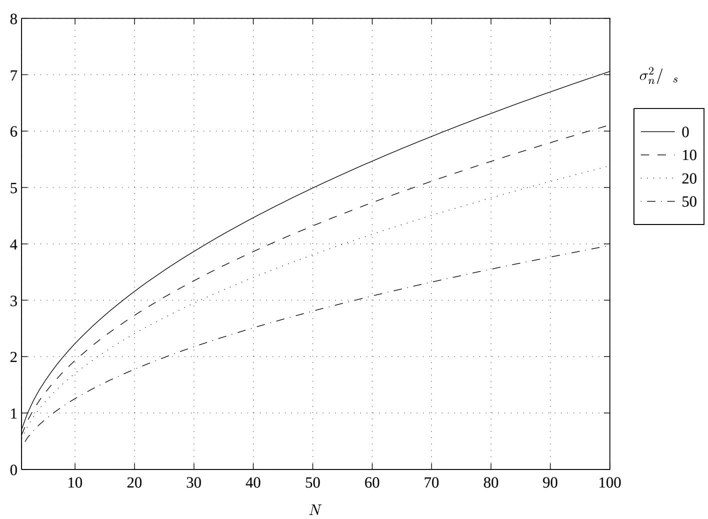

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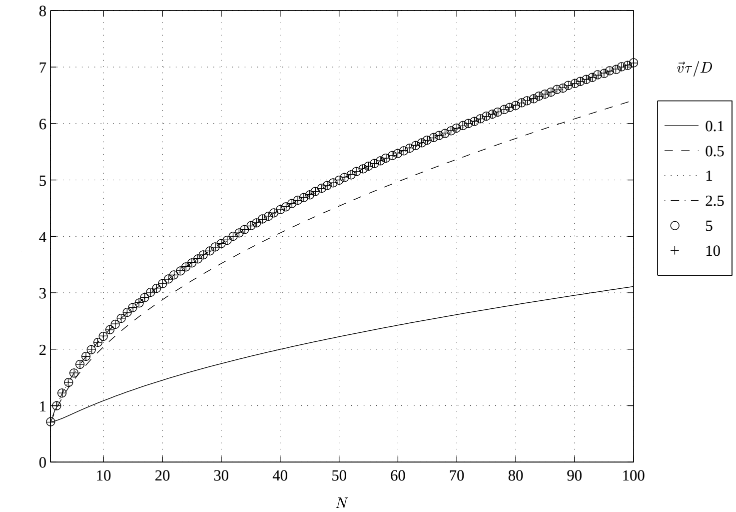

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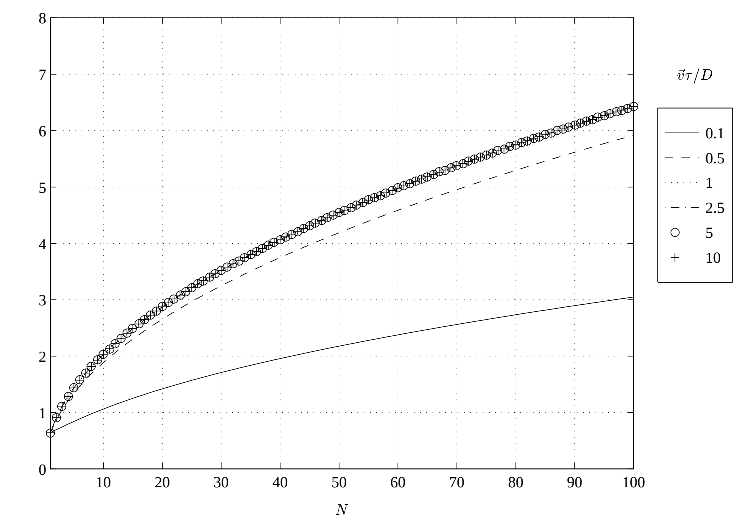

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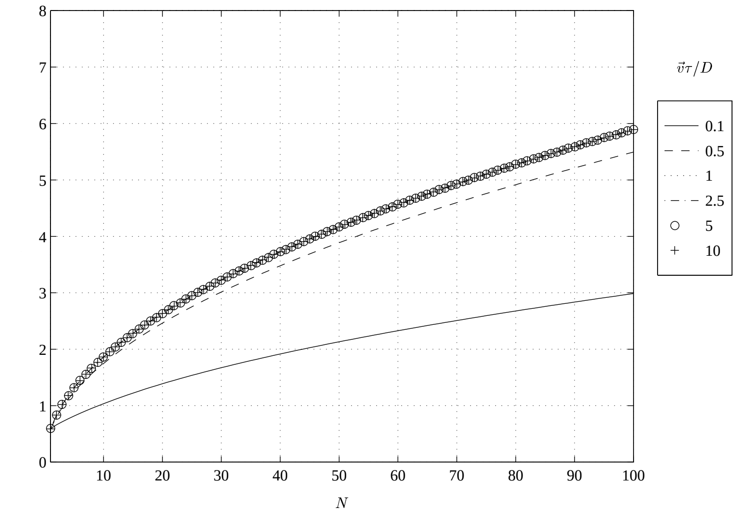

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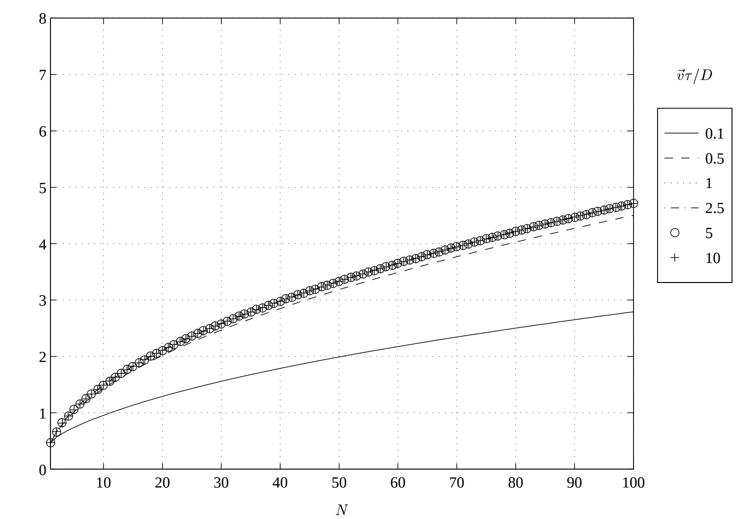

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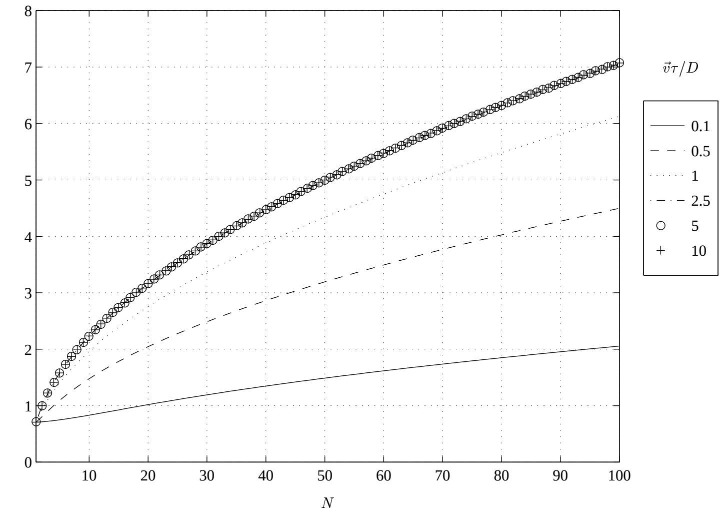

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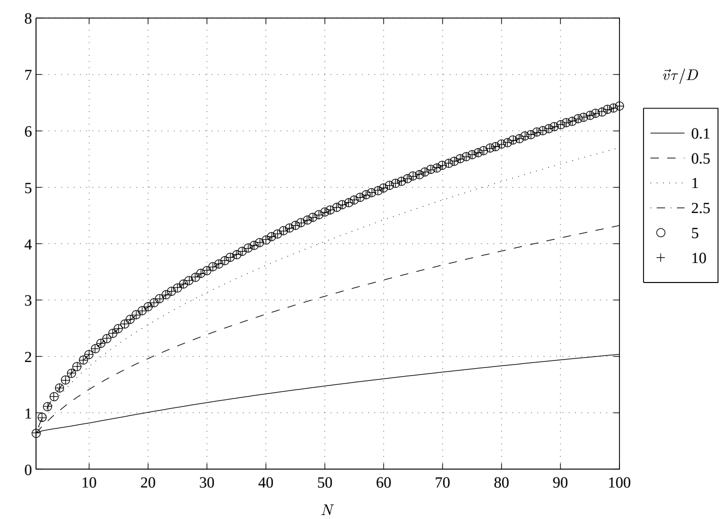

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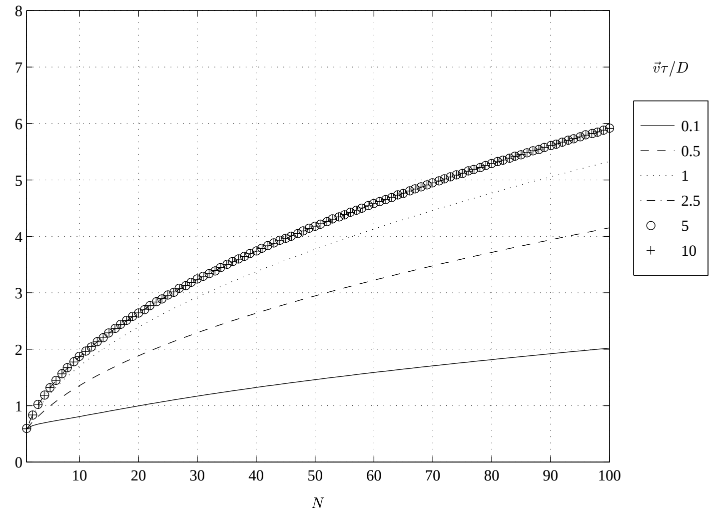

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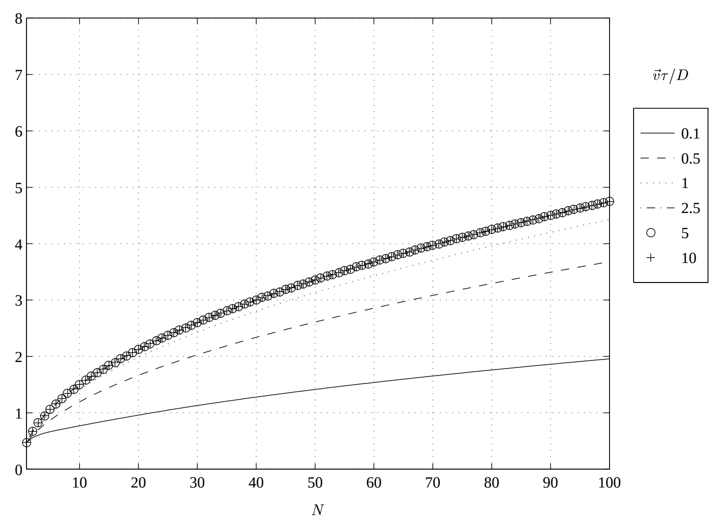

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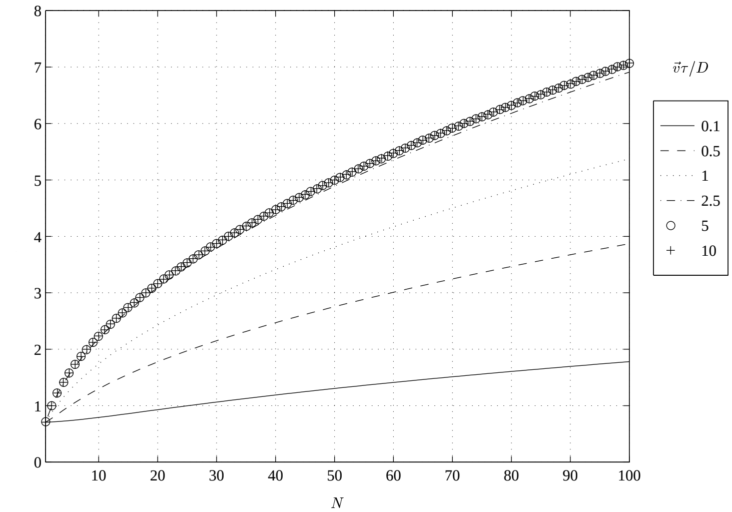

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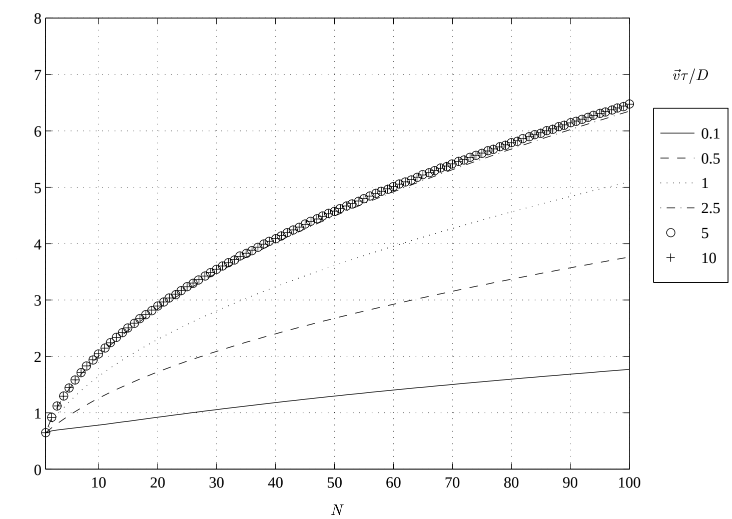

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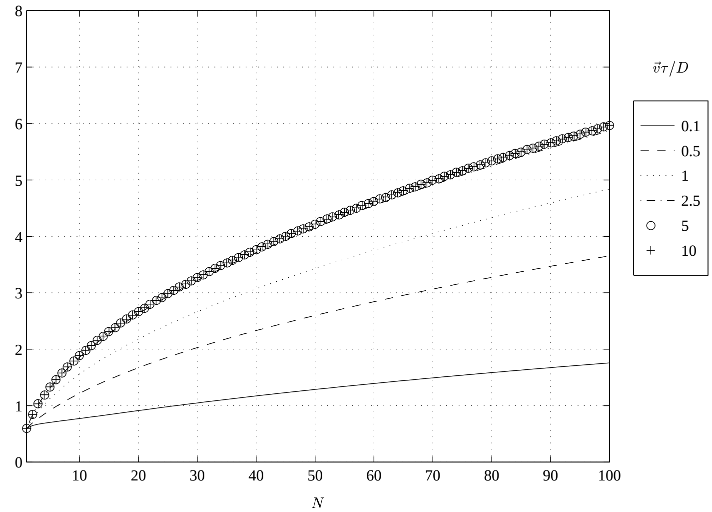

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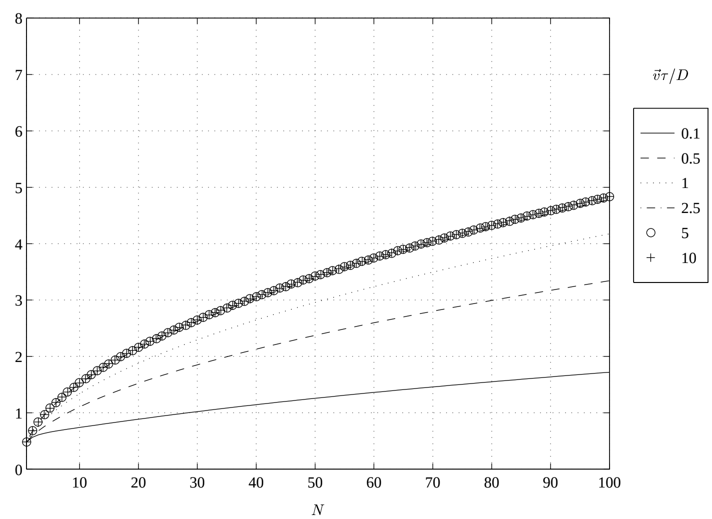

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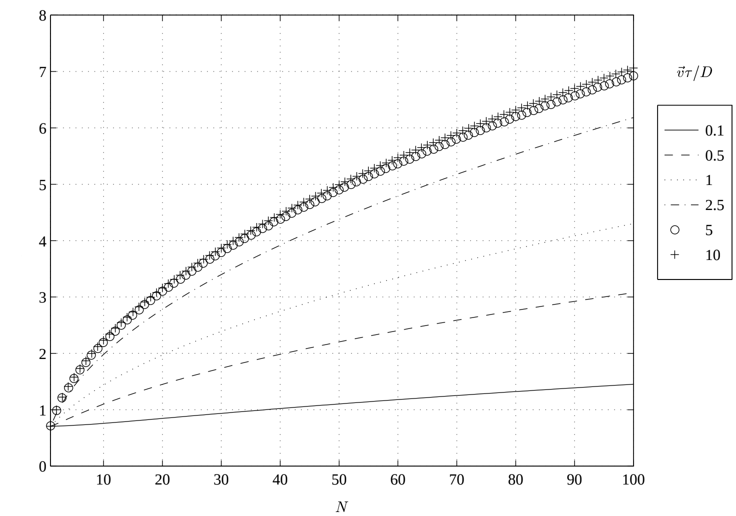

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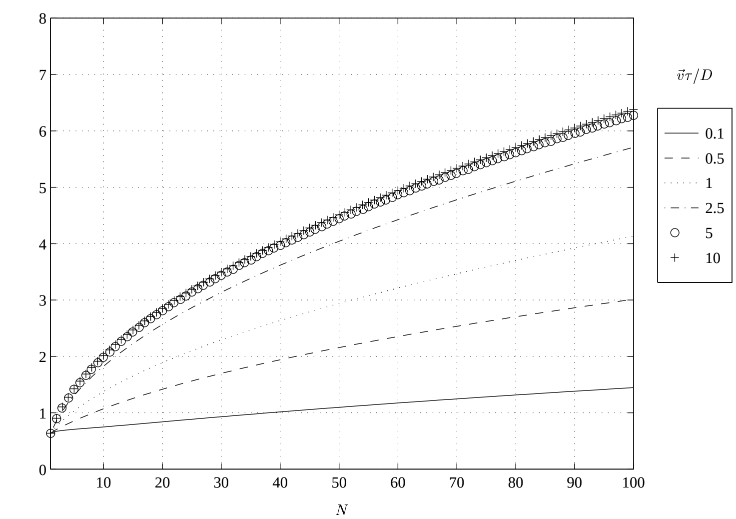

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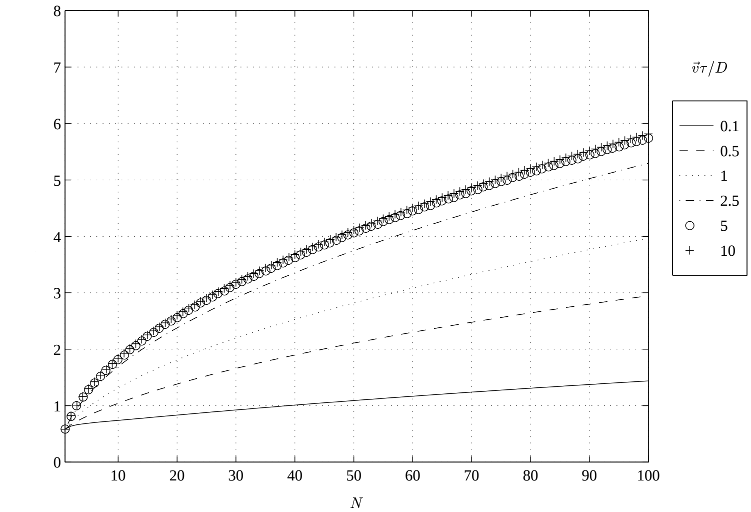

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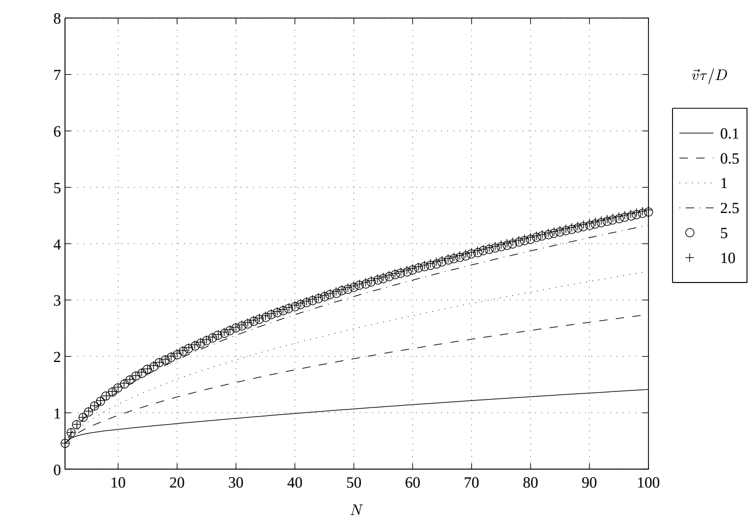

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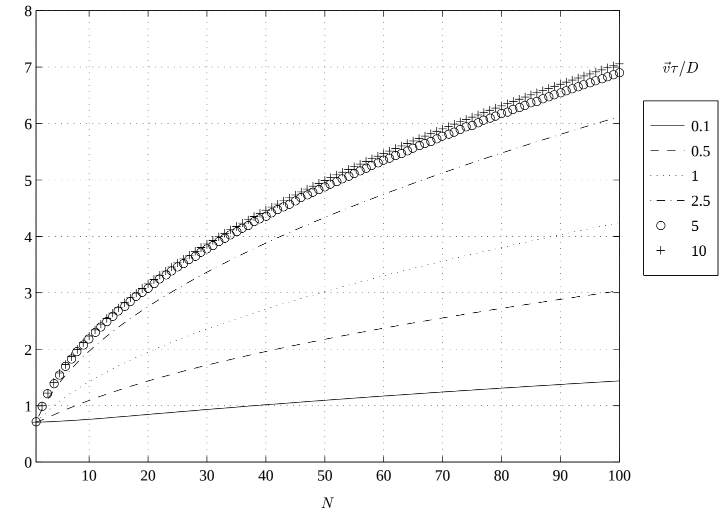

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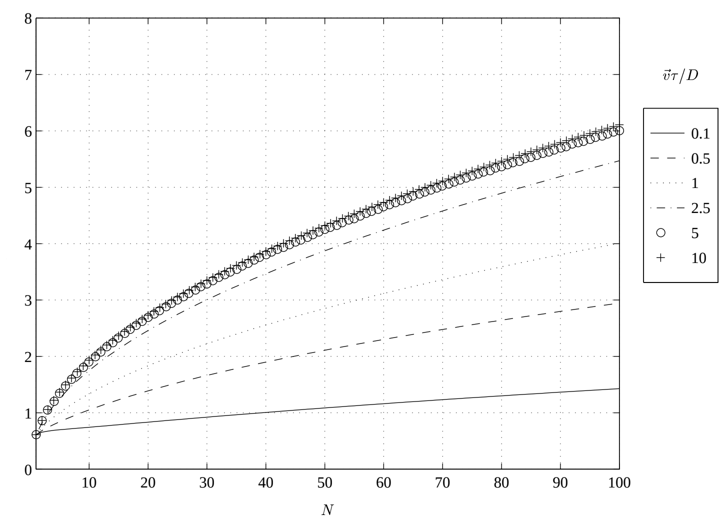

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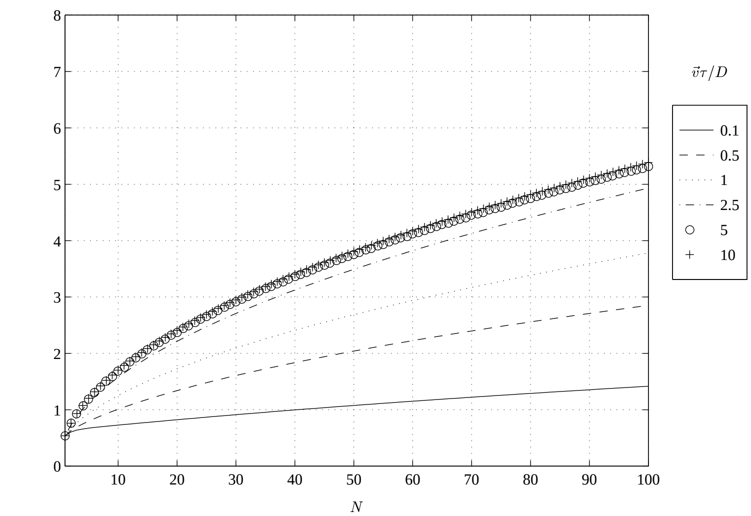

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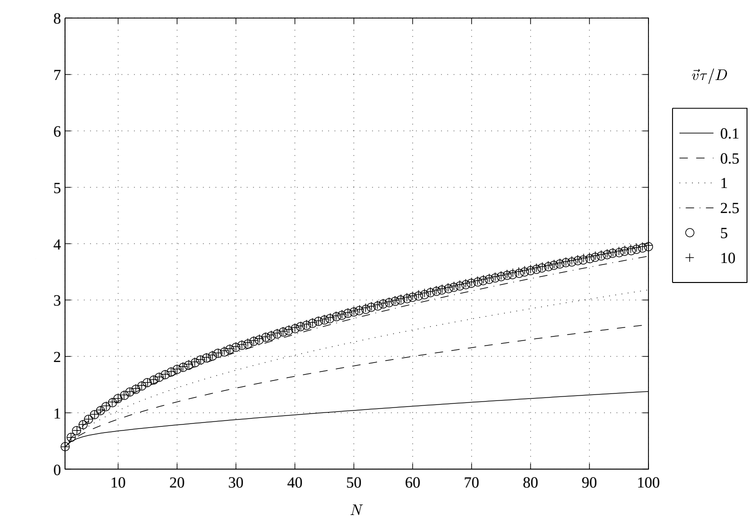

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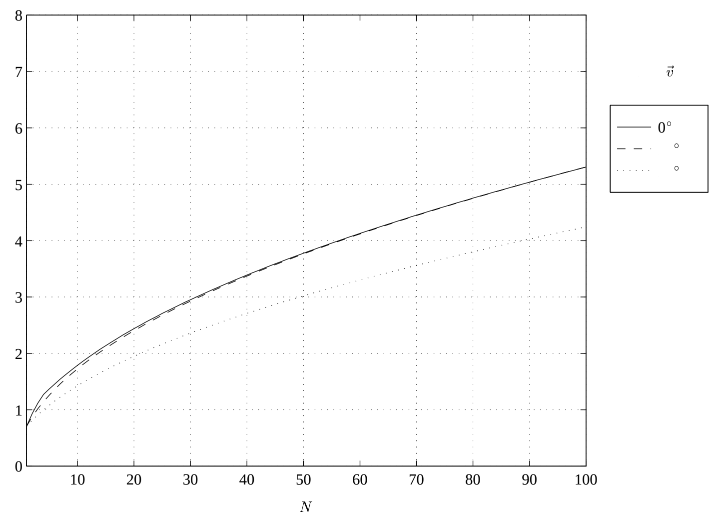

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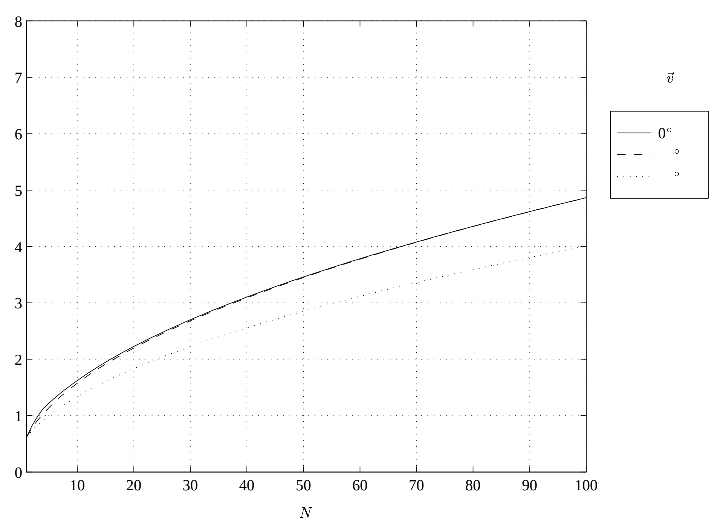

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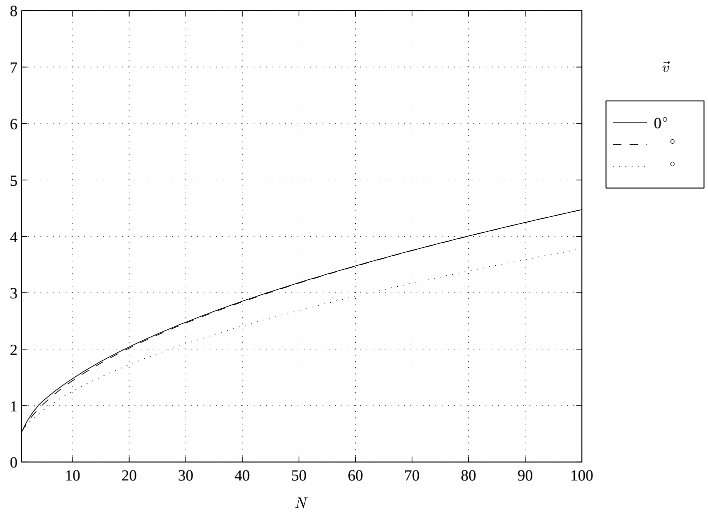

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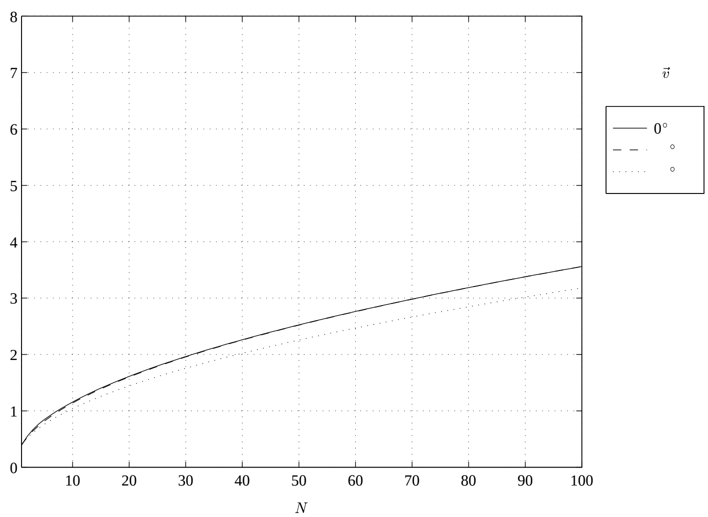

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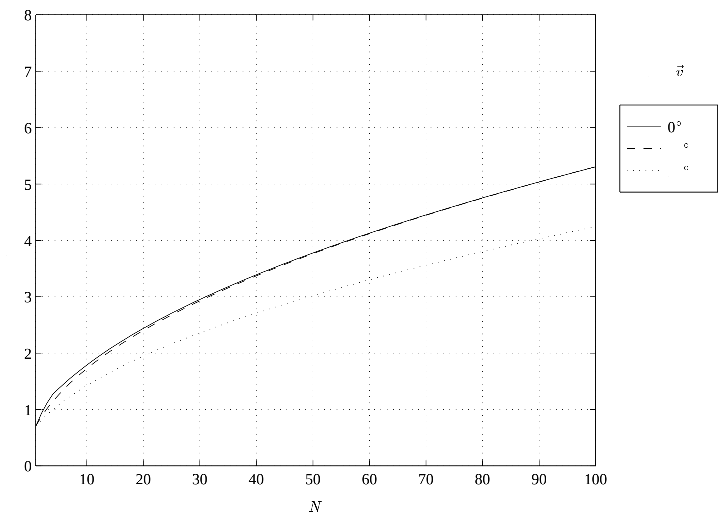

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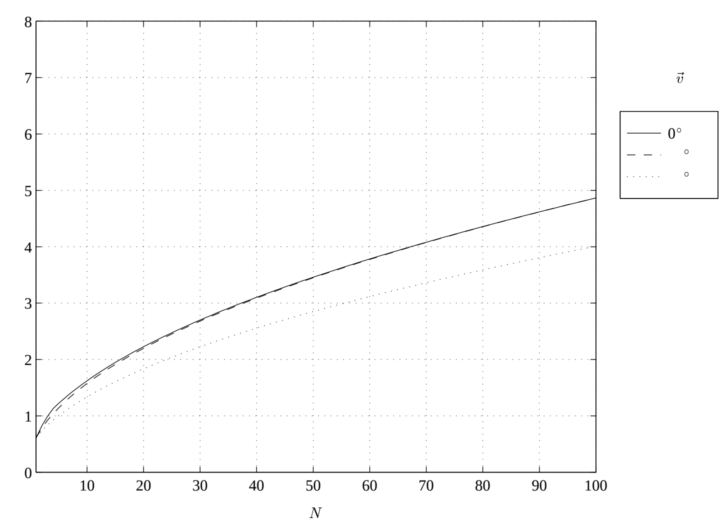

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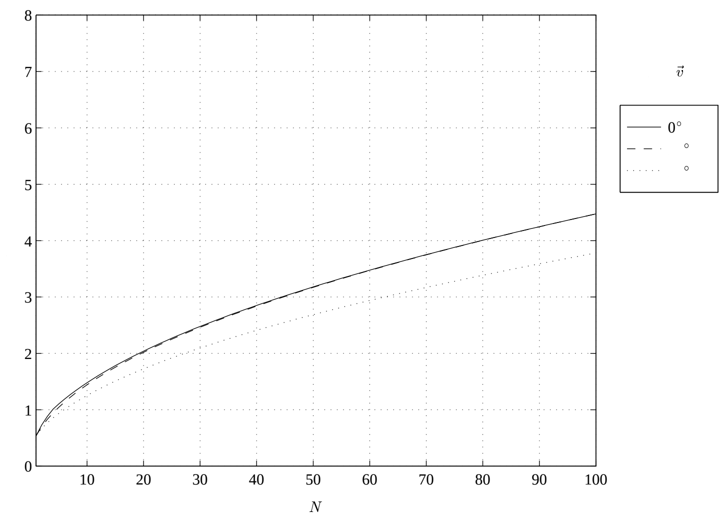

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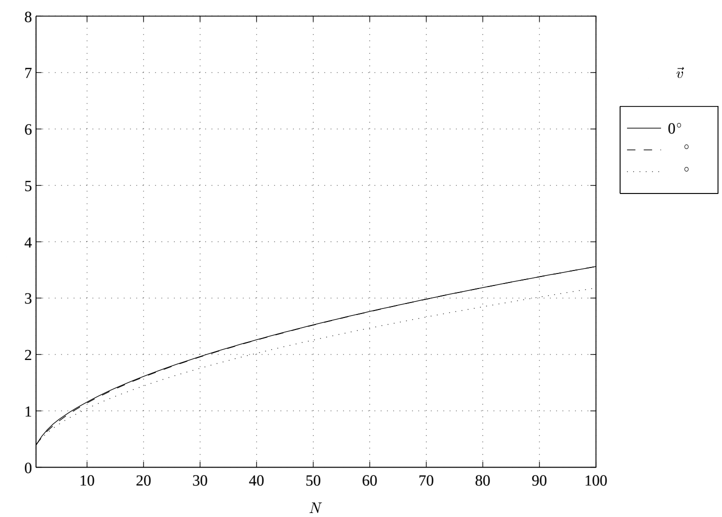

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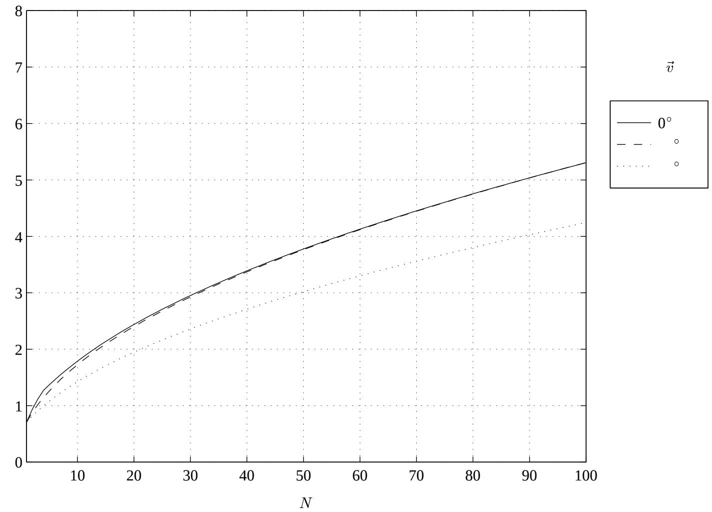

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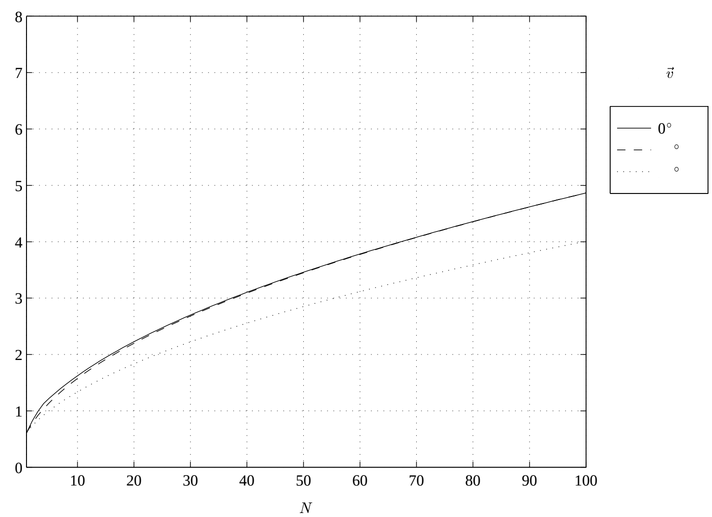

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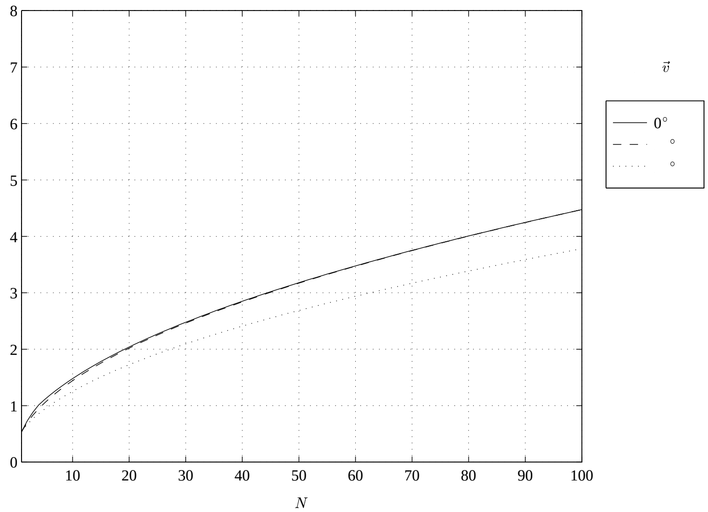

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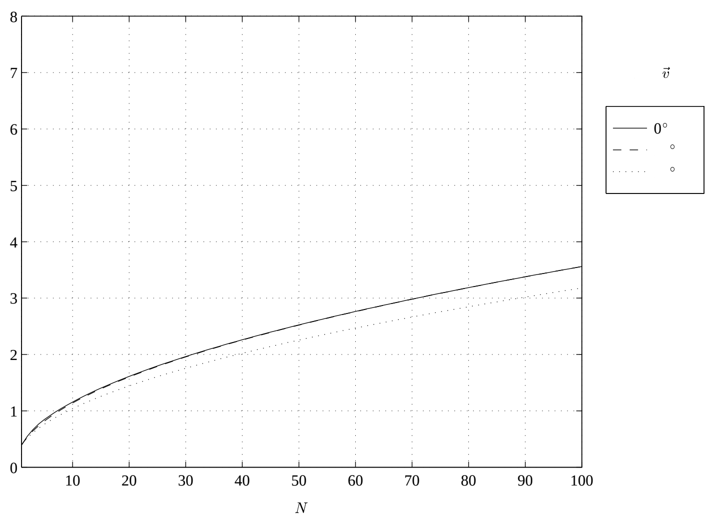

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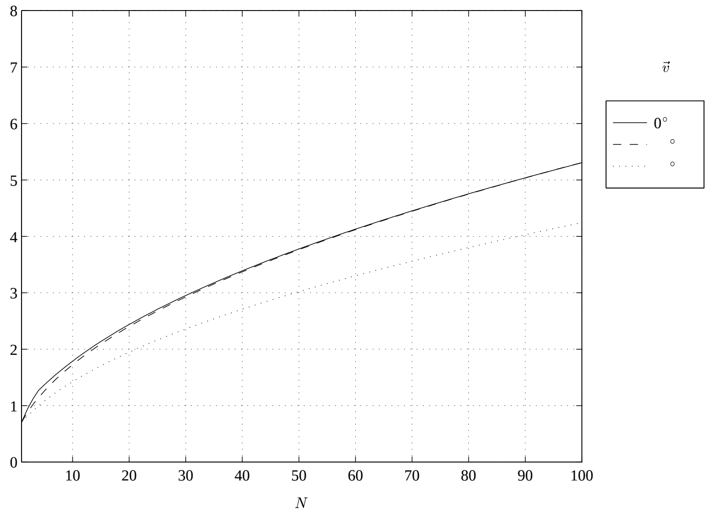

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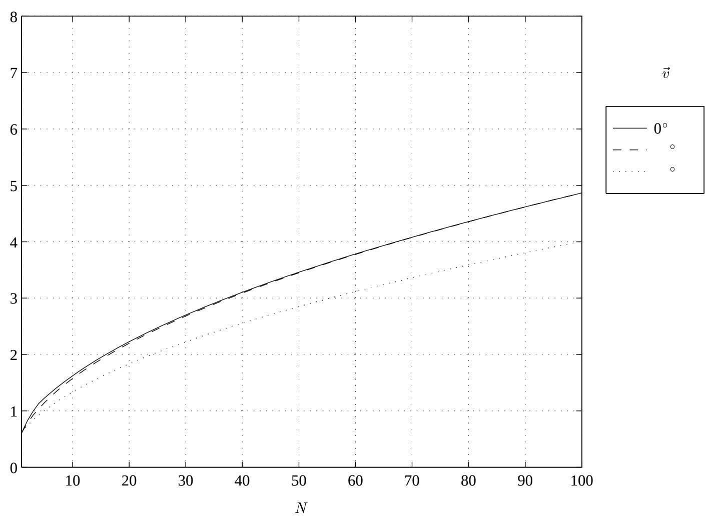

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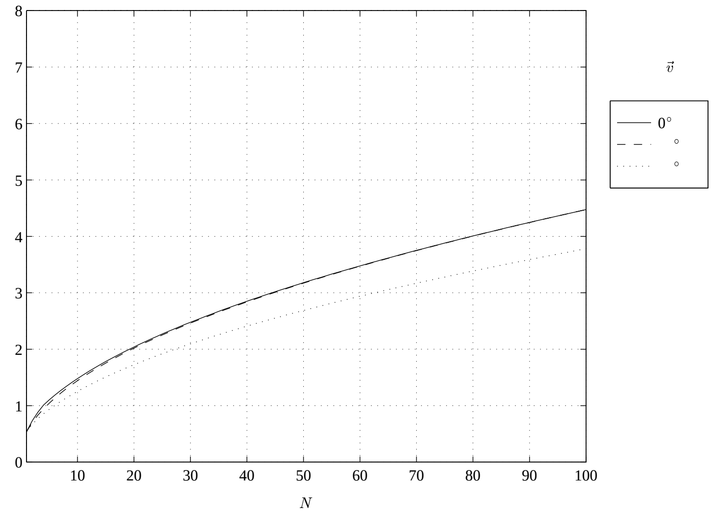

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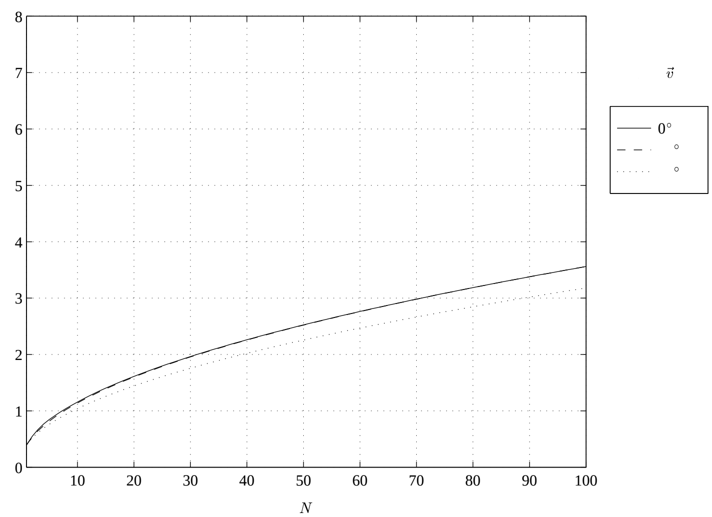

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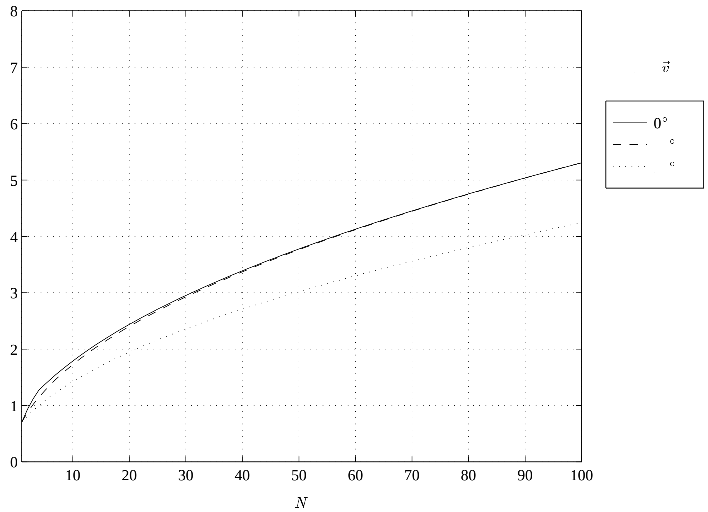

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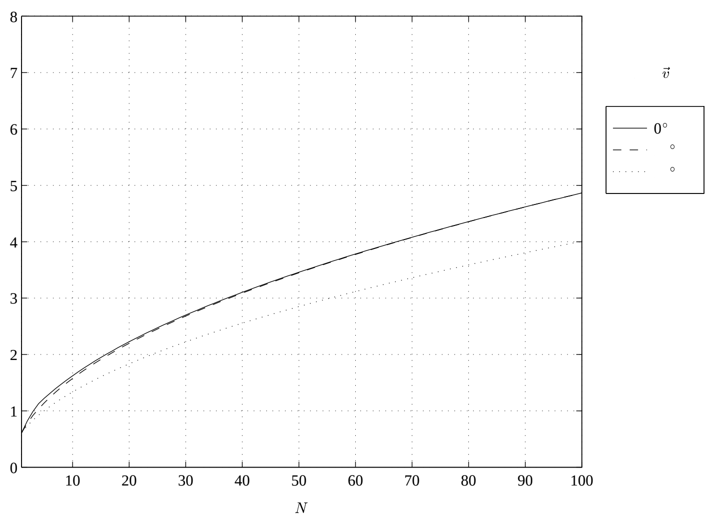

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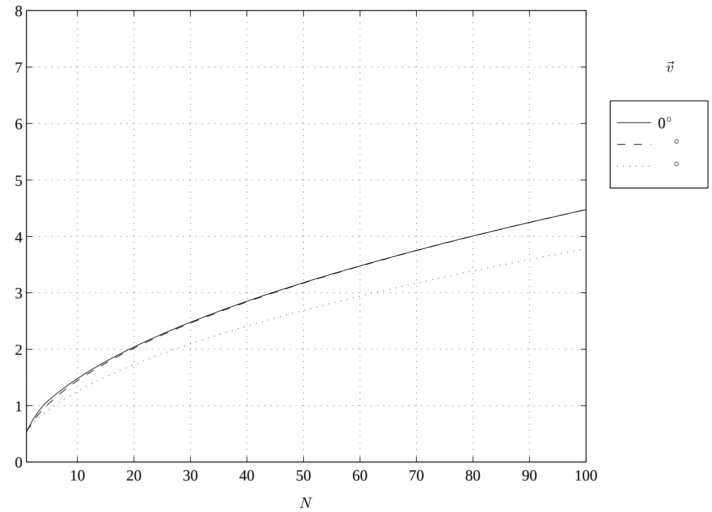

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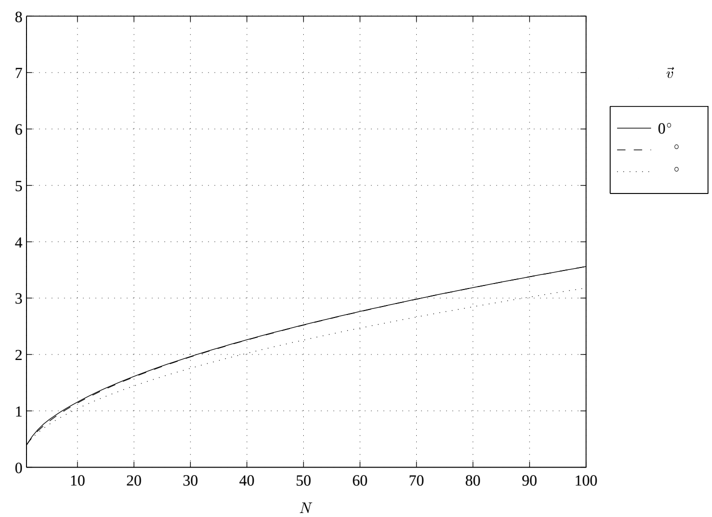

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Vita

December 1996 Master's Thesis

PERFORMANCE ANALYSIS OF A HARTMANN WAVEFRONT SENSOR USED FOR SENSING ATMOSPHERIC TURBULENCE STATISTICS

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Atmospheric turbulence parameters, such as Fried's coherence diameter, the outer scale of turbulence, and the turbulence power law, are related to the wavefront slope structure function (SSF). The SSF is defined as the second moment of the wavefront slope difference as a function of both time and position. Knowledge of the SSF allows turbulence parameters to be estimated. Hartmann wavefront sensor (H-WFS) slope measurements, composed of both signal and noise, allow the SSF to be estimated by computing a mean square difference of H-WFS slope measurements. The quality of the SSF estimate is quantified by the signal-to-noise ratio (SNR) of the estimator. This thesis develops a theoretical SNR expression for the SSF estimator. This SNR is a function of H-WFS geometry, the number of temporal frames included in the estimate, the outer scale, power law, and temporal properties of the turbulence. Spatial slope correlations are incorporated. Temporal slope correlations are incorporated using Taylor's frozen flow hypothesis. Results are presented for various H-WFS configurations and atmospheric turbulence properties.

Hartmann wavefront sensor, atmospheric turbulence sensing, signal-to-noise ratio, slope structure function, slope structure function estimator, Zernike polynomials 227

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